Using MCMC and particle filters to forecast stochastic volatility and jumps in financial time series

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Outline of the presentation:

1) **Motivation**
2) General asset price process
3) MCMC estimation method
4) Empirical application of MCMC
5) Particle filters
6) Calculating forecasts via particle filters

Motivation

- Forecasting volatility and jumps plays a crucial role in many financial applications:
  - Option pricing, VaR calculation, optimal portfolio construction, quantitative trading, etc.
- The main problem is that **volatility and jumps are unobservable**
- There are currently two classes of models for volatility and jumps estimation and modelling:
  - **a)** **Parametric approach** of Stochastic-Volatility Jump-Diffusion models (SVJD) models – estimate volatility and jumps as latent state variables with computationally intensive estimation methods such as MCMC.
  - **b)** **Non-parametric approach** using power-variation estimators – utilize high-frequency data and the asymptotic theory of power variations (Realized Variance, etc.) to construct non-parametric estimates of volatility and jumps

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Goal of our research

- Our goal is to develop tools that would enable the estimation and application of more realistic SVJD models for the modelling of asset price behavior
- The areas of study include:
  1. Incorporation of additional effects into the SVJD models, such as jump clustering - topic of **Illustration 1**, comes from Fičura and Witzany (2015a), available at: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2551807
  2. Estimation of SVJD models on intraday time series, that would account for the intraday seasonality of volatility and jump intensity – topic of **Illustration 2**, comes from Fičura and Witzany (2015b), available at: http://amse2015.cz/doc/Ficura_Witzany.pdf
  3. Utilization of high-frequency power-variation estimators for the estimation of daily SVJD models – **Illustration 3**, comes from Fičura and Witzany (2015c), available at: https://msed.vse.cz/msed_2015/article/170-Ficura-Milan-paper.pdf

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The general price process

- Let us assume that the **logarithmic price** of an asset follows a stochastic process defined by the following SDE:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + j(t)dq(t)$$

- $p(t)$ is the logarithmic price at time t
- $\mu(t)$ is the instantaneous drift rate
- $\sigma(t)$ is the instantaneous volatility
- $W(t)$ is the Wiener process
- $j(t)$ is a process determining the size of the jumps
- $q(t)$ is a process determining the occurrence of jumps
- $\lambda(t)$ may by a process determining the jump intensity
- We can directly observe only $p(t)$ at discrete points in time
- All of the other processes are **unobservable**

Stochastic volatility

- Empirical studies show that volatility is time-varying
- i.e. the term $\sigma(t)$ is following its own stochastic process
- A widely used model for $\sigma(t)$ is the **log-variance model**
- In this model the $h(t) = \ln(\sigma_t^2)$ follows a **mean-reverting Ornstein-Uhlenbeck process:**

$$dh(t) = \kappa[\theta - h(t)]dt + \xi dW_V(t)$$

- κ determines the strength of the mean reversion
- θ determines the long-term volatility, and
- ξ determines the volatility of volatility
- $W_V(t)$ is a Wiener process that may be correlated with $W(t)$
- After discretization the O-U process becomes AR(1) process

Self-Exciting jumps

- Empirical studies indicate also some form of **jump clustering**
- i.e. the **intensity of jumps is time varying** $dq(t)$
- We can model the clustering using the **self-exciting Hawkes process** (with exponential decay function) for the term
- The **jump intensity** $Pr[dq(t)=1]=\lambda(t)dt$ is then governed by the following process:

$$d\lambda(t)=\kappa[\theta-\lambda(t)]dt+\eta dq(t)$$

- By solving the equation we can get the value of $\lambda(t)$

$$\lambda(t)=\theta+\int_{-\infty}^{t}\eta e^{-\kappa(t-s)}dq(s)=\theta+\sum_{dq(s)=1,s\leq t}\eta e^{-\kappa(t-s)}$$

Daily returns and variablity

- Assuming the general process for log-price evolution:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + j(t)dq(t)$$

- **Daily returns** $r(t) = p(t) - p(t-1)$ can then be expressed:

$$r(t) = \int_{t-1}^{t} \mu(\tau)d\tau + \int_{t-1}^{t} \sigma(\tau)dW(\tau) + \sum_{t-1 \leq \tau < t} \kappa(\tau)$$

- The variability of the price process can be expressed with its **quadratic variation** in the form:

$$QV(t) = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{t-1 \leq s < t} \kappa^2(s)$$

- Which is a sum a of **integrated variance** and **jump volatility:** $QV(t) = IV(t) + JV(t)$

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Bayesian estimation methods

- First we have to define the assumed stochastic processes for the **logarithmic price**, **stochastic volatility**, **jumps**, etc.
- Bayesian methods can then be used to estimate the process **parameters** and the values of the **latent state variables**
- i.e. the values of the latent **stochastic volatilities**, **jump occurrences**, **jumps sizes**, etc. for every single day in the time series
- Commonly used methods are:
  - **Markov Chain Monte Carlo (MCMC)**
  - **Particle filters (PF)**

Markov Chain Monte Carlo

- **MCMC** algorithm allows us to **sample from multivariate distributions** by constructing a Markov chain
- It can be used to estimate **model parameters** and **latent state variables** by approximating their **joint posterior density**
- <u>Many versions of the algorithm exist:</u>
  - **Gibbs sampler** – Uses conditional densities to estimate joint density
  - **Metropolis-Hastings algorithm** – Uses rejection sampling
    - **Random-walk** – Uses only the likelihood ratio
    - **Multiple-step** – Similar to Random-Walk but with faster convergence
    - **Independence sampling** – Using approximate densities
- The methods can be combined, using different methods for different variables

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Illustration 1 - The SVJD model

- Model described in Fičura and Witzany (2015a)
- We define our **jump-diffusion model with stochastic volatility and self exciting jumps** as follows:

- Log-ret:

$$dp(t) = \mu dt + \sigma(t)dW(t) + j(t)dq(t)$$

$$j(t) \sim N(\mu_J, \sigma_J)$$

- Stoch.vol:

$$dh(t) = \kappa[\theta - h(t)]dt + \xi dW_V(t)$$

$$h(t) = \ln(\sigma_t^2)$$

- Intenzity:

$$d\lambda(t) = \kappa_J\left[\theta_J - \lambda(t)\right]dt + \eta_J dq(t)$$

$$Pr[dq(t) = 1] = \lambda(t)dt$$

Euler discretization

- We use the **Euler discretization** with the assumption that **at most 1 jump can happen during one day**
- The discrete model has the following equations:

$$r(t) = \mu + \sigma(t)\varepsilon(t) + J(t)Q(t)$$

$$h(t) = \alpha + \beta h(t-1) + \gamma\varepsilon_V(t)$$

$$\lambda(t) = \alpha_J + \beta_J\lambda(t-1) + \gamma_J Q(t-1)$$

$$\alpha = (1-\beta)\theta \qquad \alpha_J = (1-\beta_J)\theta_J$$

$$\varepsilon(t) \sim N(0,1) \qquad \varepsilon_V(t) \sim N(0,1)$$

$$V(t) = \sigma_t^2$$

$$h(t) = \ln(V(t))$$

$$Q(t) \sim \mathrm{Bern}(\lambda(t))$$

$$J(t) \sim N(\mu_J, \sigma_J)$$

- 9 parameters are being estimated: $\mu, \alpha, \beta, \gamma, \theta_J, \beta_J, \gamma_J, \mu_J, \sigma_J$
- And 3 series of latent state variables: ***V,J,Q***

MCMC algorithm

- We estimate the **model parameters** and the **latent state variables** using **MCMC algorithm**
- The **MCMC algorithm** combines:
  - **Gibbs Sampler** $(\mu, \sigma, \mu_j, \sigma_j, \alpha, \beta, \gamma, \mathbf{Q}, \boldsymbol{J})$
  - **Accept-Reject Gibbs Sampler** (***V***)
  - **Random Walk Metropolis-Hastings** $(\theta_J, \beta_J, \gamma_J)$
- The estimation algorithm was firstly **tested on simulated time series** with **mixed results** in its **ability to identify jumps** and their clustering
- Especially if σ_j in the simulated data was not high enough, the jumps mixed with the diffusion volatility and the estimated θ_J was much lower then in the simulations
- We further show only the results for the real data (EUR/USD time series)

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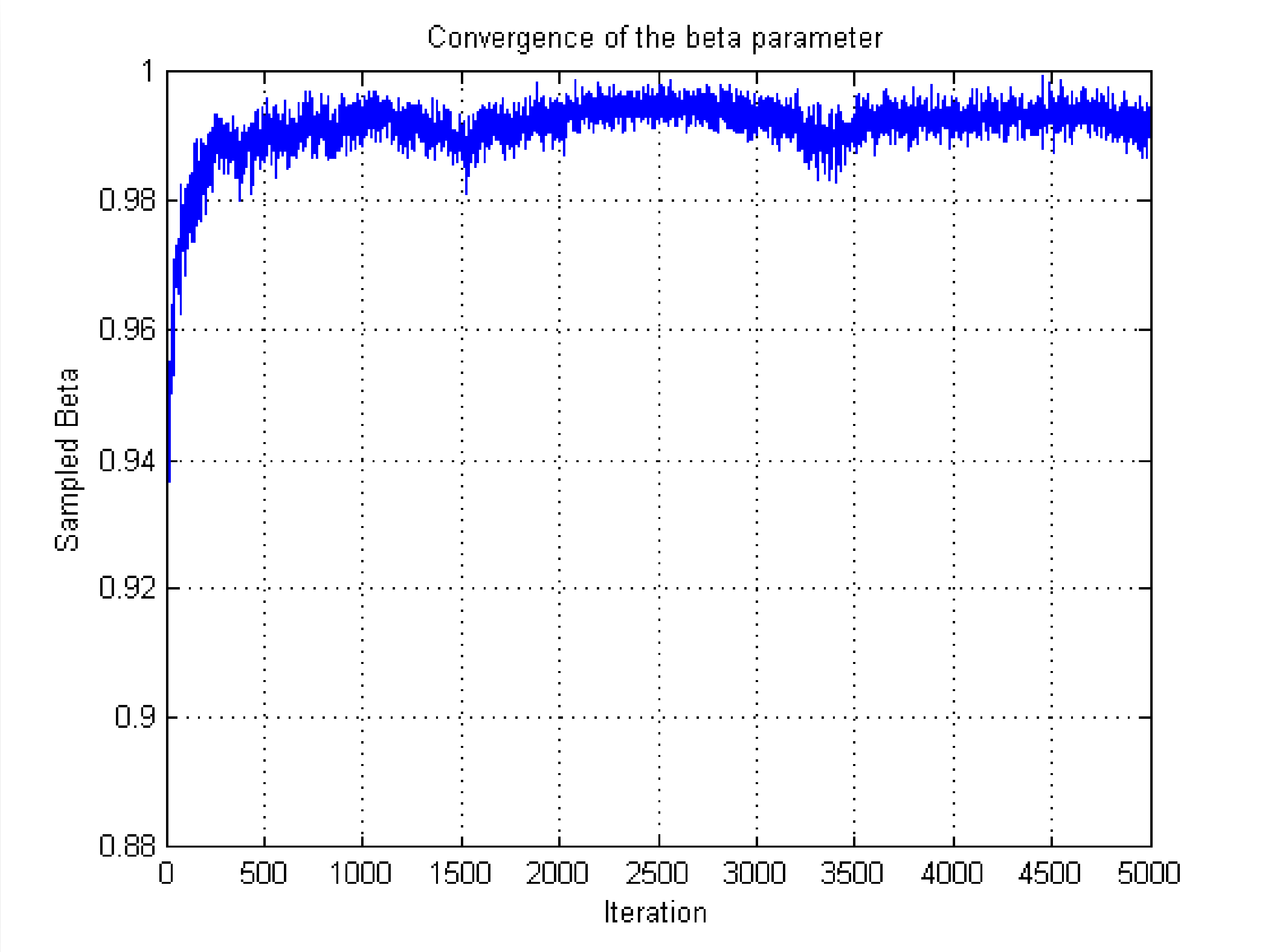
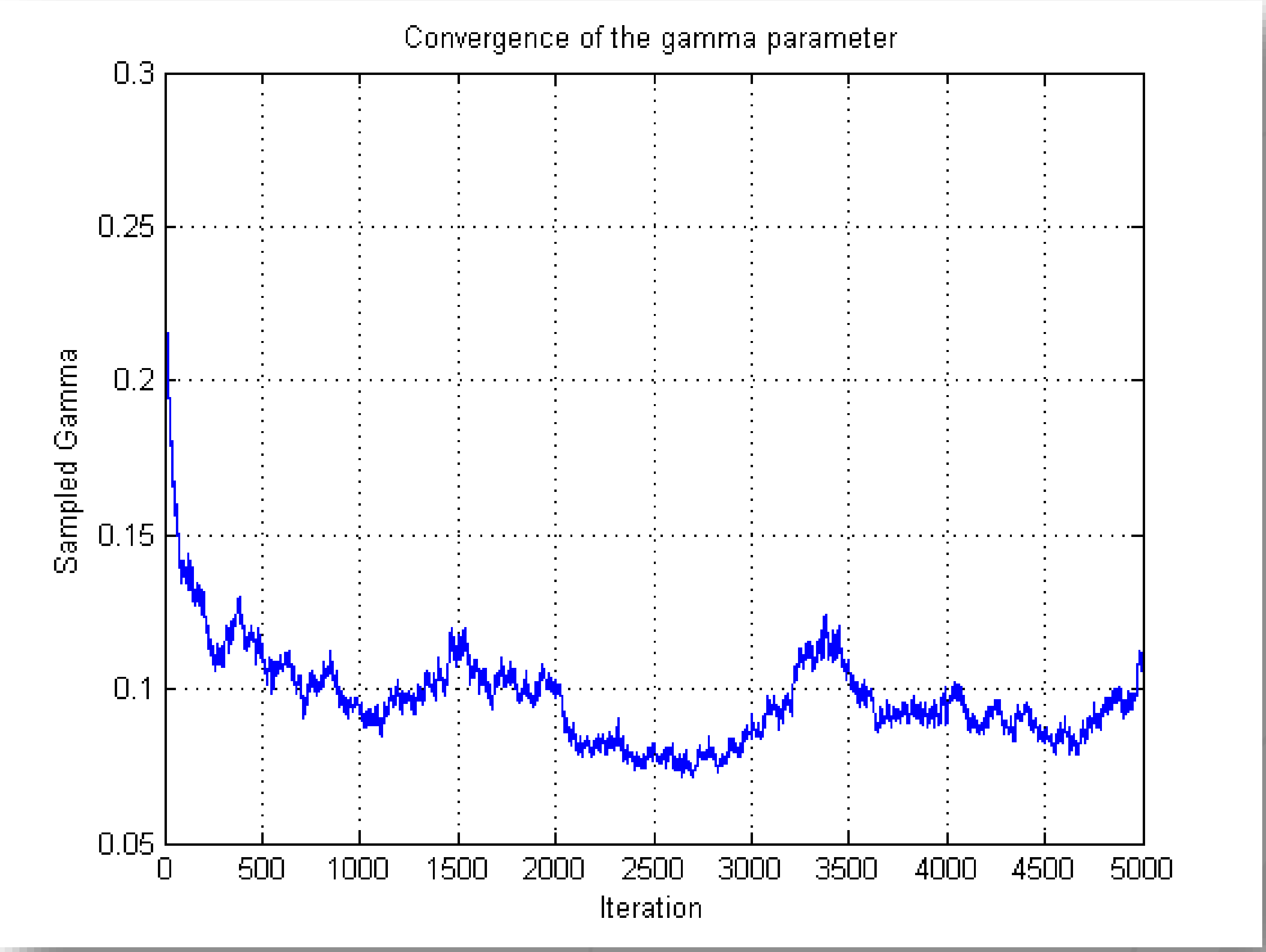
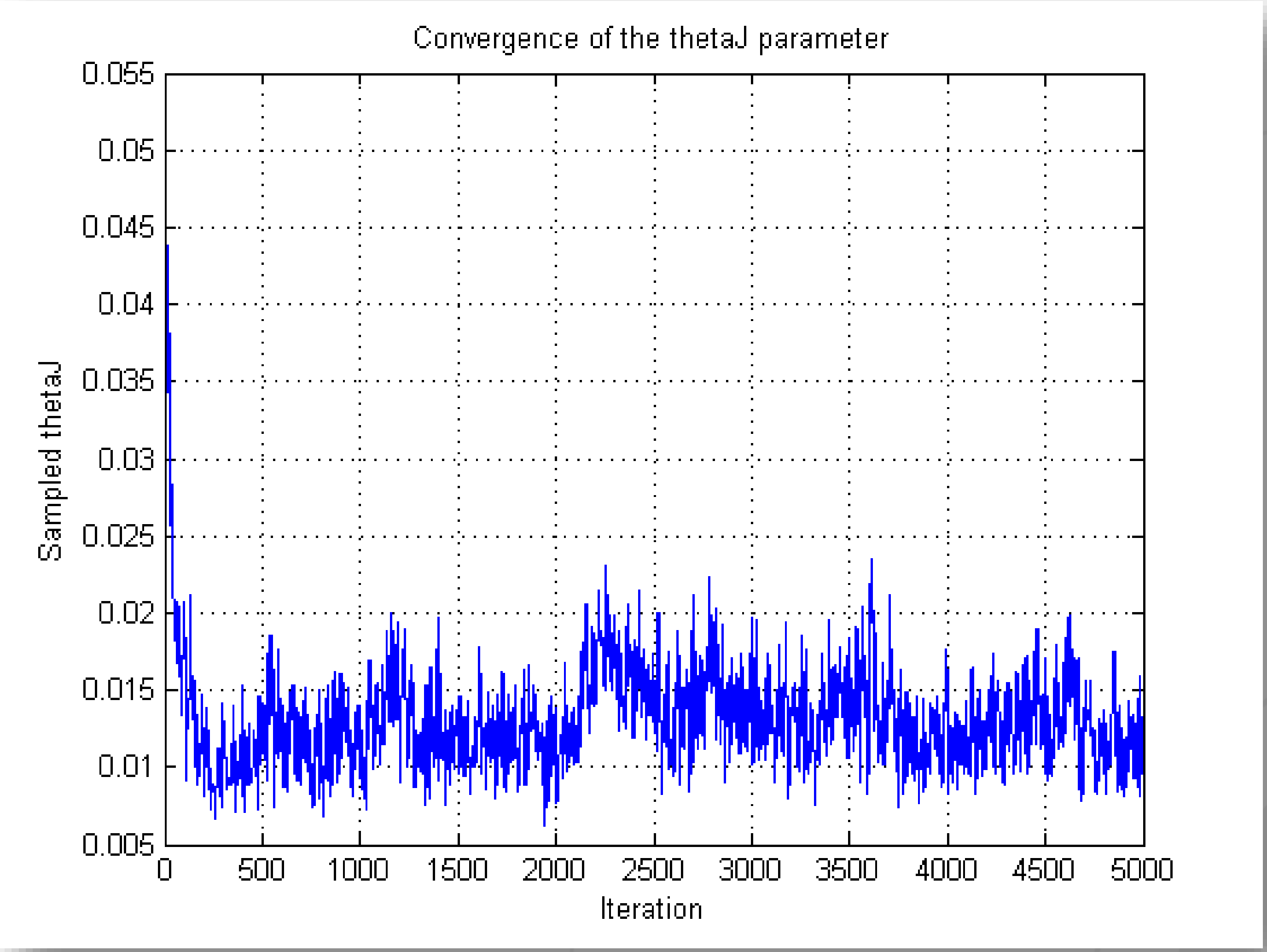
Convergence of Beta



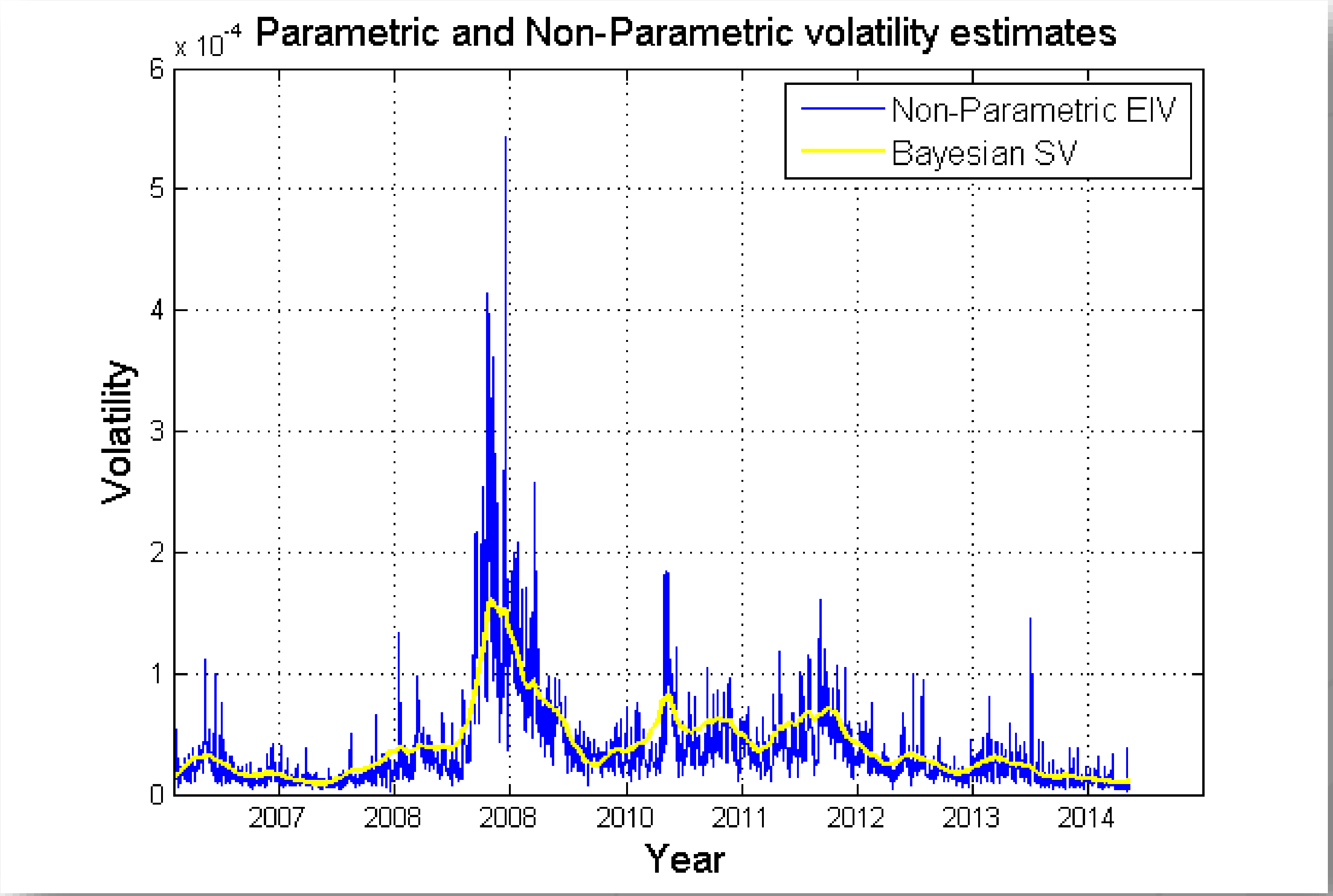
Convergence of Gamma



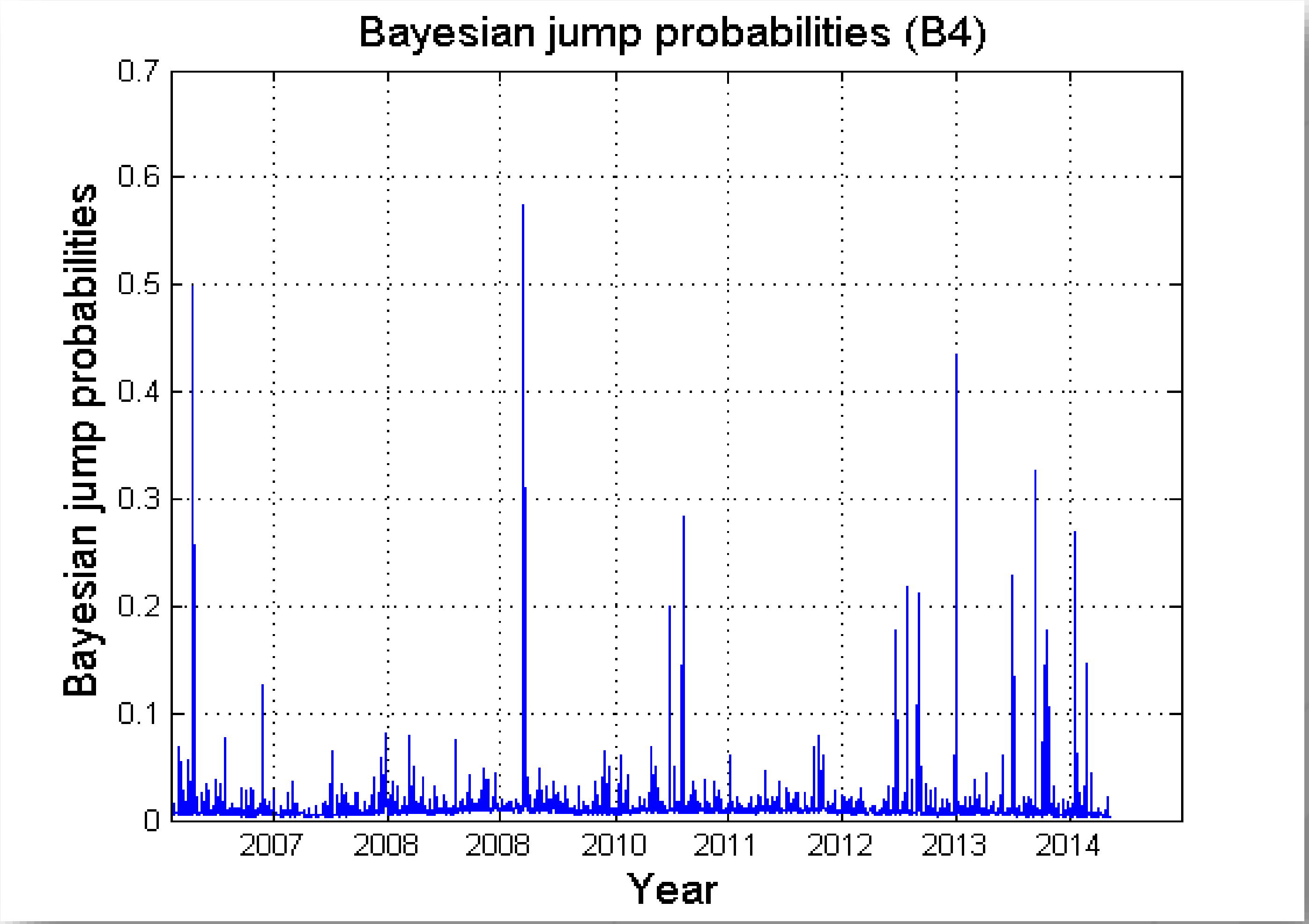
Convergence of ThetaJ



Realized vs. estimated variance



Bayesian jump probabilities



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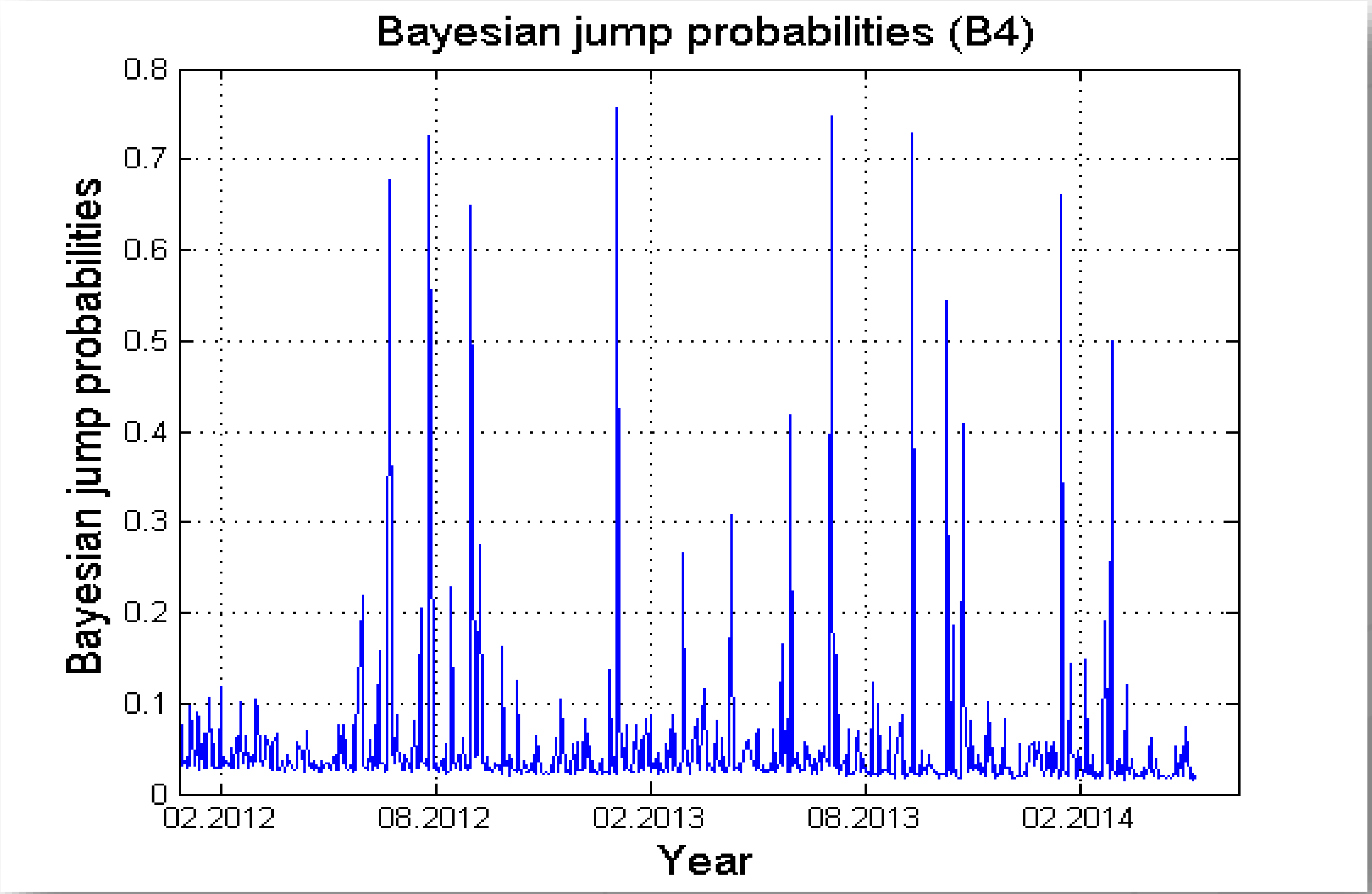
Jump prob. (since 2012)



Can the model identify jumps?

- The performance of the model in this regard does not look very good – **too few jumps**
- Also the **self-exciting** property does **not seem to be present** (the mode of betaJ and gammaJ distributions is close to zero)
- The **jumps identified** using **the bayesian model** were further compared with the ones identified using the **shrinkage estimator (Z-Statistics):**
  - **A.** **Mean probabilities of jumps** for every single day were calculated using **their bayesian distributions**
  - B. The daily **probabilities of jumps** were calculated from the **shrinkage estimator** using the standard normal CDF
- **Spearman rank correlation coefficient** was calculated between the two variables with value of only **<u>0,0171</u>**
- i.e. the **probabilities of jumps** estimated using the two different methods do **not seem to be rank correlated**

Ilustration 2 – Intraday SVJD

- Extended model was used in Fičura and Witzany (2015b), to model volatility and jumps based on intraday price returns (4-hour returns specifically)
- It was necessary to incorporate intraday seasonality of the volatility and jump intensity into the model
- The model contains:
  - 9 parameters related to the stochastic processes
  - 10 parameters associated with the seasonality effects: s_j and $\lambda_{S,j}$ with j going from 1 to 5 (as one in the six seasons is chosen as benchmark with parameter value equal to one)
  - Three vectors of latent state variables: **V, J** and **Q**
- The estimation is performed by using a MCMC algorithm combining **Gibbs sampler** and **Metropolis-Hastings algorithm**

Full intraday SVJD model:

- The **logarithmic returns** are given as:

$$r(t) = \mu + \sigma(t)\varepsilon(t) + J(t)Q(t)$$

- The **stochastic volatility** is given as:

$$\sigma(t) = v(t)s(t)$$

$$h(t) = \alpha + \beta h(t-1) + \gamma \varepsilon_V(t)$$

$$s(t) = \sum_{j=1}^{6} s_j d_j(t)$$

- The **jump intensity** is given as:

$$\lambda(t) = \lambda_H(t)\lambda_S(t)$$

$$\lambda_H(t) = \alpha_J + \beta_J \lambda_H(t-1) + \gamma_J Q(t-1)$$

$$\lambda_S(t) = \sum_{j=1}^{6} \lambda_{S,j} d_j(t)$$

$$Q(t) \sim Bern[\lambda(t)]$$

$$J(t) \sim N(\mu_J, \sigma_J)$$

$$\varepsilon(t) \sim N(0,1)$$

$$h(t) = \log(V(t))$$

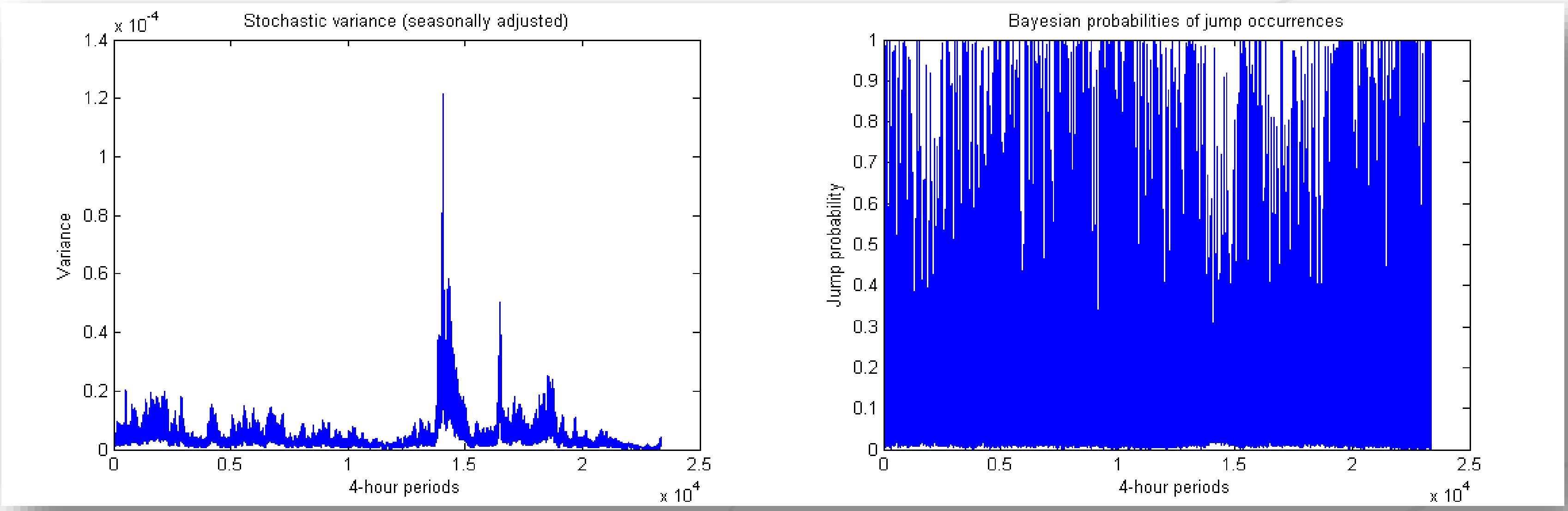
$$V(t) = v^2(t)$$

$$\varepsilon_V(t) \sim N(0,1)$$

$$Pr[Q(t) = 1] = \lambda(t)$$

Latent state variable estimates

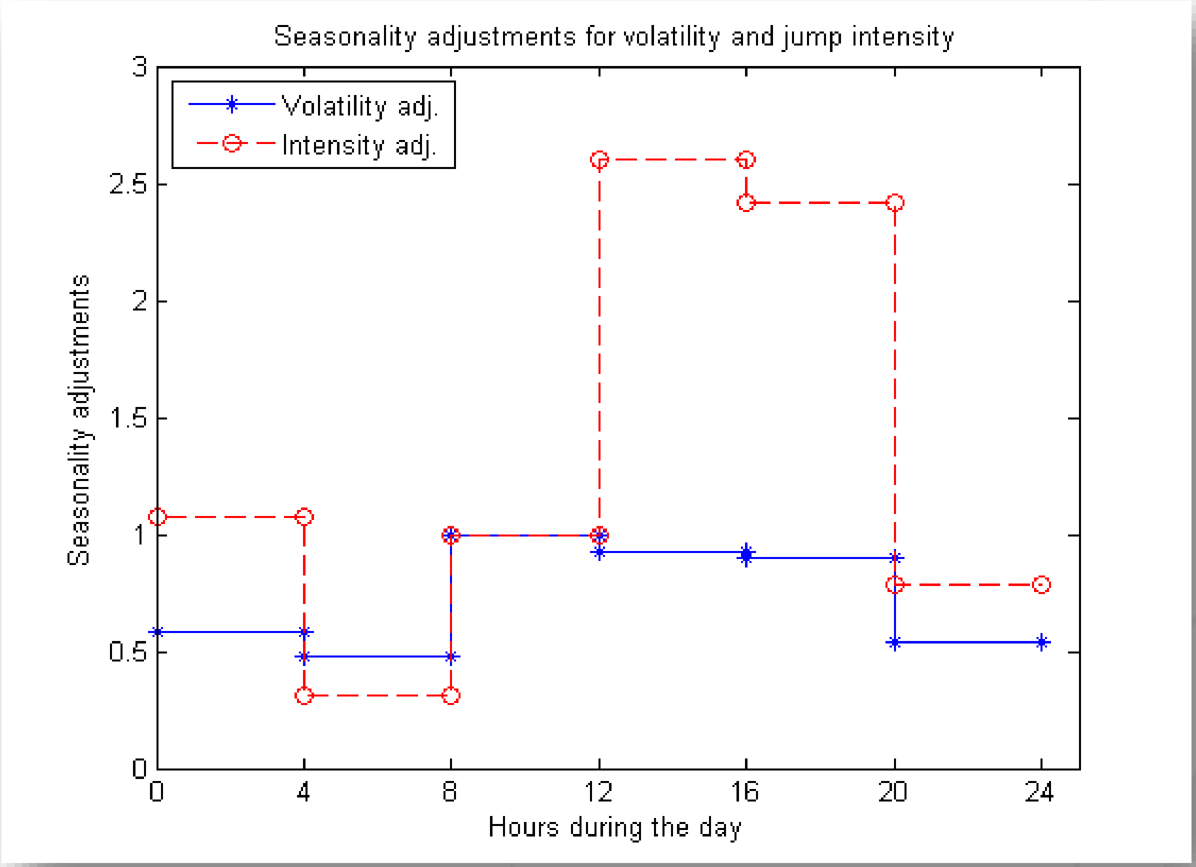
- Posterior mean estimates of the stochastic variances and jump occurrences:



- Range of tests confirmed that the estimates of volatility and jumps based on the intraday SVJD model correspond more closely to the non-parametric estimates of these quantities then when a daily SVJD model is used

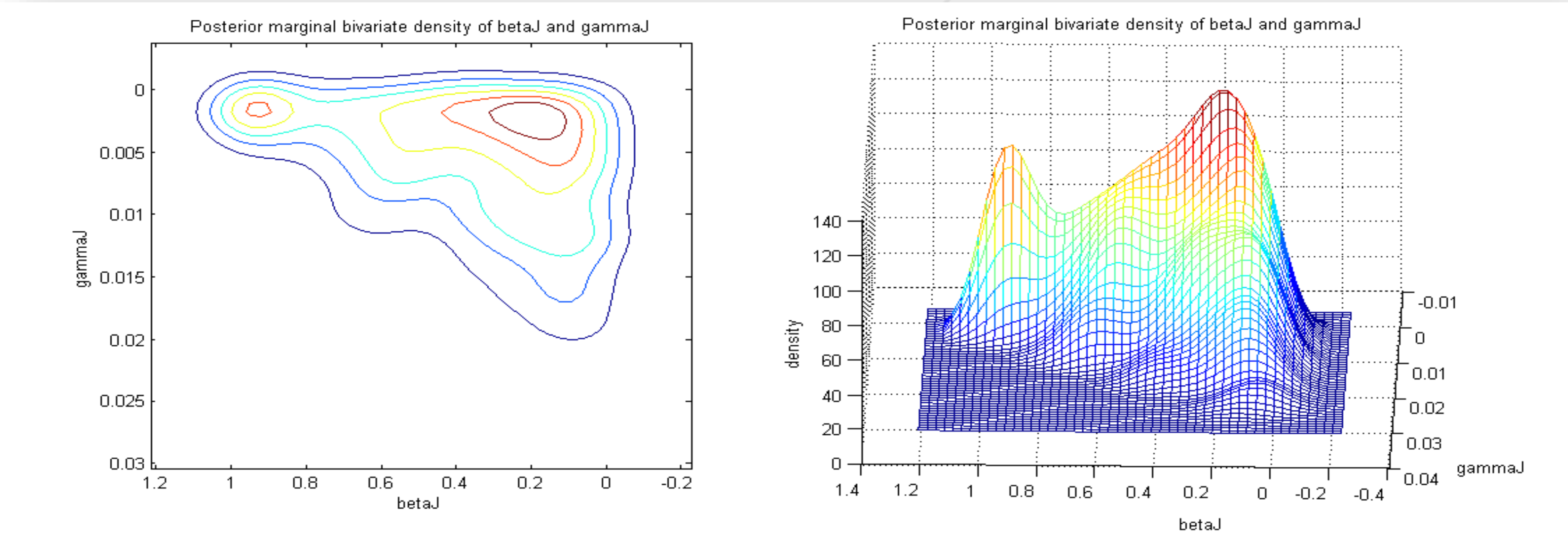
Intraday seasonality adjustments

- The model identified the following intraday seasonality patterns of volatility & jump intensity



Posterior marginal distribution of betaJ and gammaJ

- The estimated posterior marginal bivariate distribution of betaJ and gammaJ exhibited bimodality



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Particle filters

- **Particle filters** use weighted set of particles and Bayesian recursion equations in order to estimate the posterior density over a set of latent state-space variables
- **Differences compared to MCMC:**
- With MCMC we were estimating $p(V_t|\mathcal{F}_T)$
- Where $\mathcal{F}_T$ denotes the information over the whole history of the time series
- With particle filters we are estimating $p(V_t|\mathcal{F}_t)$
- Where $\mathcal{F}_t$ denotes the observable information until time *t*
- After estimating $p(V_t|\mathcal{F}_t)$, we can make forecasts of $p(V_{t+1}|\mathcal{F}_t)$, $p(V_{t+2}|\mathcal{F}_t)$, etc. via simulations
- The models can thus be used for volatility forecasting, VaR estimation, Option pricing, etc.

Illustration 3 – SVJD-RV-Z

- High-frequency power-variation estimators can be utilized to get additional information for the bayesian estimation of the SVJD models (Fičura and Witzany, 2015c)
- **SVJD-RV –** Model uses realized variance together with daily returns in order to estimate the stochastic volatilities
- **SVJD-RV-Z** – Model uses also the Z-Estimator of jumps in order to more accurately estimate jumps in the time series
- The SVJD-RV-Z model is constructed so that it can distinguish between small jumps (visible only on the intraday frequency) and large jumps (influencing the returns on the daily frequency)
- Models were applied to the EUR/USD exchange rate evolution in the period between 1.11.1999 and 10.10.2014 containing a total of 3 884 trading days

The SVJD-RV-Z model

- The **logarithmic returns** are given as:

$$r(t) = \mu + \sigma(t)\varepsilon(t) + J(t)Q(t)$$

- The **stochastic volatility** is given as:

$$h(t) = \alpha + \beta h(t-1) + \gamma\varepsilon_V(t)$$

- The **jump intensity** is given as:

$$\lambda(t) = \alpha_J + \beta_J\lambda(t-1) + \gamma_J Q(t-1)$$

- The **realized variance** is given as:

$$\log\left(RV(t) - J^2(t)Q(t)\right) = h(t) + \varepsilon_{RV}(t)$$

- The **Z statistics** is given as:

$$Z(t) = \mu_Z + \xi_Z Q(t) + \varepsilon_Z$$

$$\varepsilon(t) \sim N(0,1)$$

$$J(t) \sim N(\mu_J, \sigma_J)$$

$$Q(t) \sim Bern[\lambda(t)]$$

$$h(t) = \log(V(t))$$

$$V(t) = \sigma^2(t)$$

$$\varepsilon_V(t) \sim N(0,1)$$

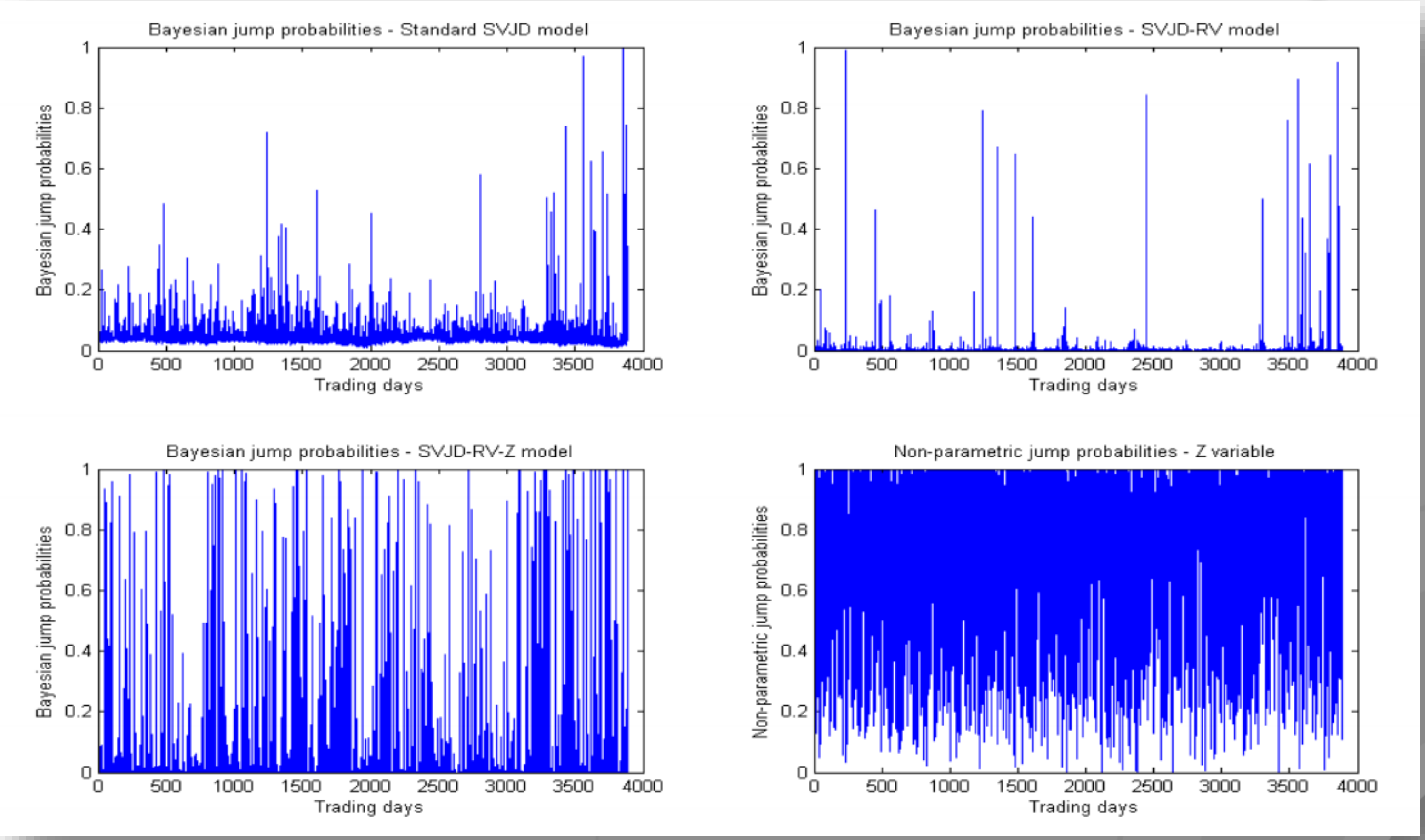
$$Pr[Q(t) = 1] = \lambda(t)$$

$$\varepsilon_{RV}(t) \sim N(0, \sigma_{\mathrm{RV}})$$

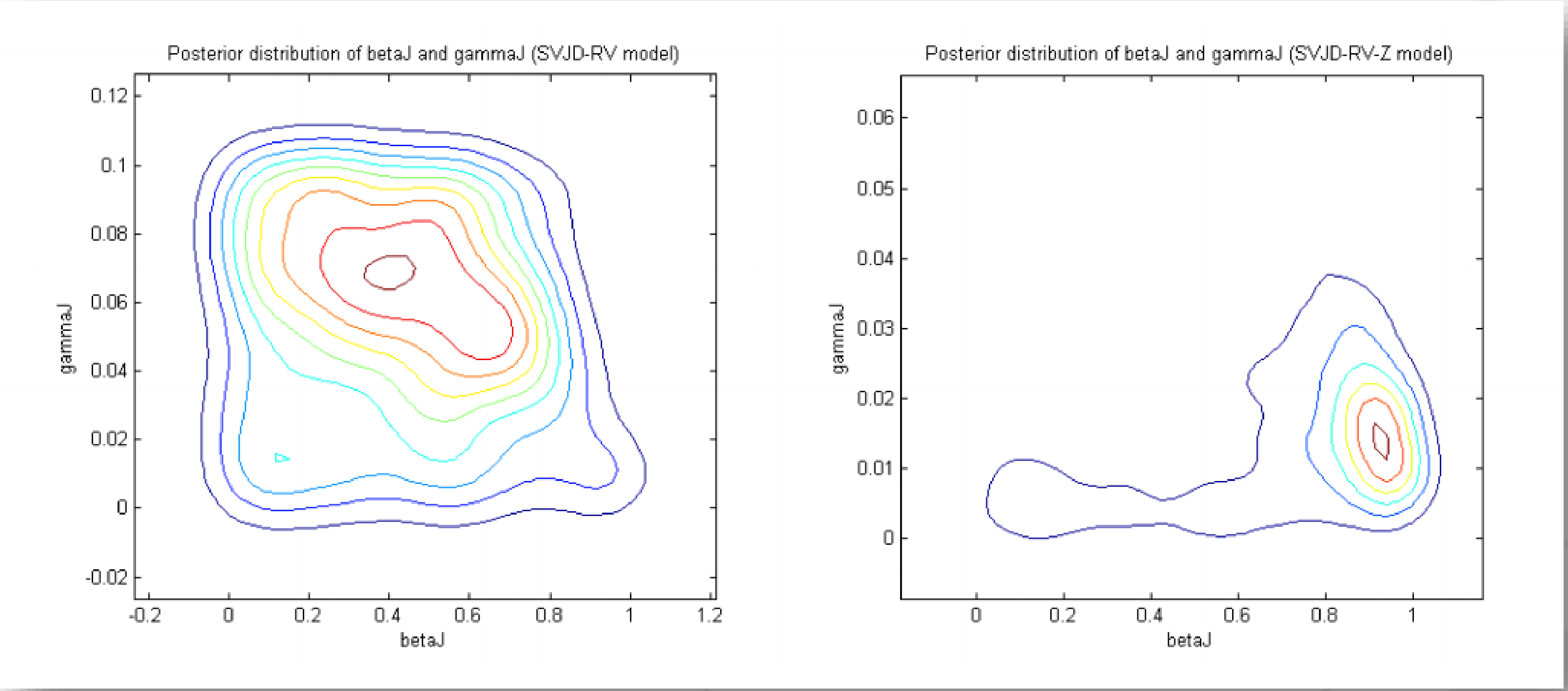
$$\varepsilon_Z \sim N(0, \sigma_Z)$$

Jump estimates - 4 models

- Estimated jumps under SVJD, SVJD-RV, SVJD-RV-Z (with MCMC) and with the Z-Statistics only

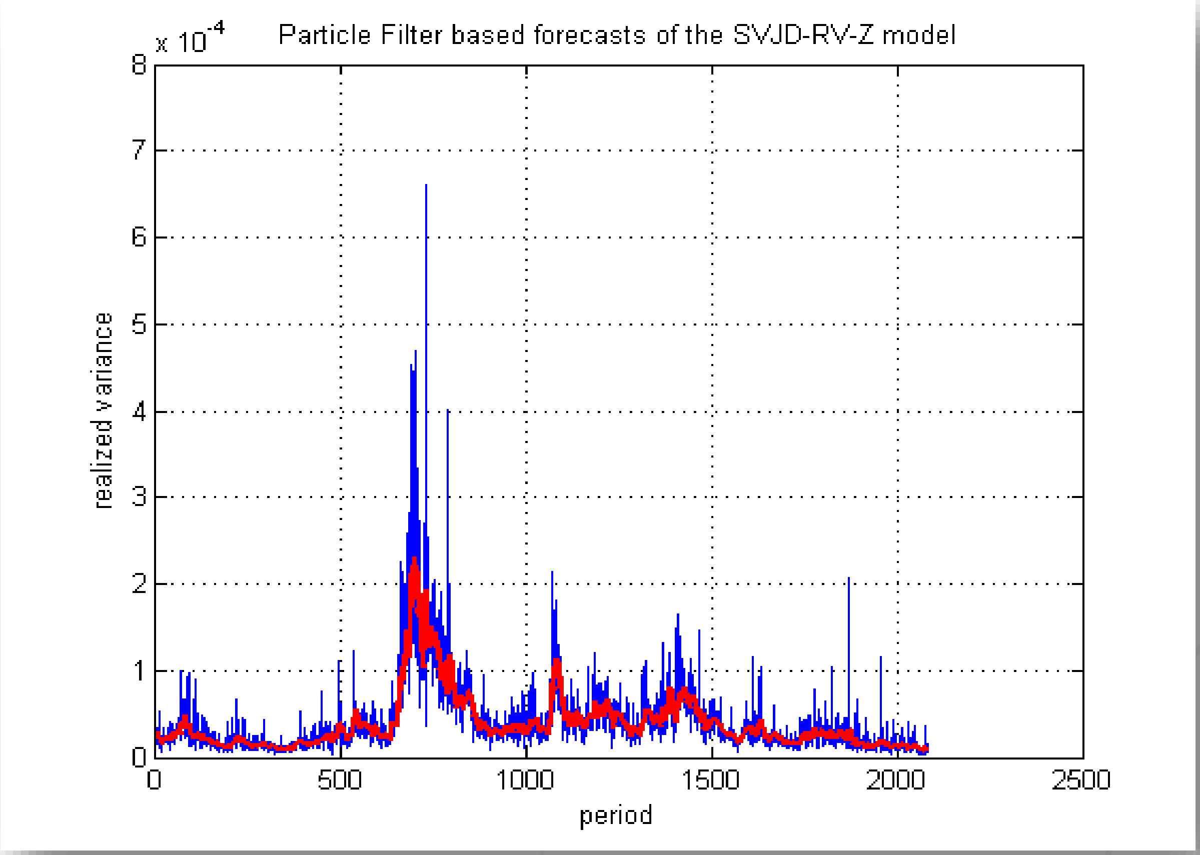


Marginal density of GammaJ and BetaJ (for SVJD-RV and SVJR-RV-Z)



Particle Filter forecasts

- One period ahead forecasts of the SVJD-RV-Z model constructed by using the SIR particle filter



Preliminary results – Out-Sample forecasts

- The out-of-sample R-Squared values are comparable with the HAR model, which is commonly used as benchmark.
- Models estimated with MCMC on the first 1000 periods and applied via particle filters to the rest of the data

Horizon	SV	SV-RV	SVJD	SVJD-RV	SVJD-RV-Z	SVJDH	SVJDH-RV	SVJDH-RV-Z	HAR
1 Day	0.3982	0.4577	0.3867	0.4593	0.4648	0.3923	0.4621	0.4652	0.4746
5 Day	0.5556	0.6523	0.5633	0.6564	0.6539	0.5598	0.6562	0.6628	0.6551
20 Day	0.5825	0.6969	0.5902	0.6986	0.7005	0.5795	0.6955	0.7029	0.7064

- Further research:
  - We will add jumps in the volatility process
  - We will add long-memory to the volatility process
  - Auxiliary particle filter instead of SIR filter
  - Leverage effect in order to model stock market volatility
  - Compare with more advanced modifications of HAR

Conclusions

- The estimation of SVJD models on intraday returns as well as the utilization of intraday power-variation estimators both lead to more realistic jump estimates
- These estimates include significantly more pronounced self-exciting effects of the jumps
- Out-Sample analysis of the models shows that they posses predictive power similar to the HAR model
- As our currently used SVJD models are in principle short-memory, while HAR is a long-memory model, it is expected that the incorporation of long-memory features into the SVJD models may increase the predictive power further
- Similarly the inclusion of volatility jumps and other effects may prove to be useful

Thank you for your attention

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