Debt, Inflation and Growth:

Robust Estimation of Long-Run Effects in Dynamic Panel Data Models[1]

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[1] The views expressed in this paper are those of the authors and do not necessarily represent those of Federal Reserve Bank of Dallas, the Federal Reserve System, the International Monetary Fund or IMF policy.

Issues and problems

- There is a renewed interest in the possible adverse effects of rising public debt on real output, in the wake of the recent global financial crisis, the sovereign debt crisis in Europe, and the protracted European growth slowdown.
- Although the long-run relationship between high and rising public debt and economic growth is an important policy question, there is little empirical work investigating the debt-growth nexus, especially using recent developments in the analysis of dynamic heterogenous panels with multi-factor error structure.

- Almost all empirical analysis of debt-growth relationship adopt strong homogeneity assumptions across countries, and are estimated without adequate attention to dynamics, feed-back effects from debt to GDP, and error cross-sectional dependencies.
- The real consequences of large debts and high inflation levels differ between the short run and the "*long term*".
- When government deficits are financed through money creation rather than issuance of debt, deficit financing could result in higher inflation rather than higher debt/GDP. To control for such cases we also consider the joint impacts of debt and inflation on growth.

- **Cross-country differences matter**. Debt-inflation-growth relationship depends on countries' characteristics such as the level of economic development; institutions; ability to generate primary fiscal surpluses and service debt; cost of borrowing; history of meeting debt obligations; vulnerabilities to shocks; and nature of investor base. *Most of these factors show up as unobserved factors.*
- **Cross-country dependencies matter:** global and regional shocks tend to affect many countries simultaneously, and generate dependencies across countries. Neglecting such dependencies can lead to biased estimates and spurious inference.
- **Nonlinearities matter**. Cross-country experience shows that some economies have run into debt difficulties and experienced subdued growth at relatively low debt levels, while others have been able to sustain high levels of indebtedness for prolonged periods and grew strongly without experiencing debt distress. Are there threshold effects? If yes, do such effects differ during rising as opposed to falling debt/GDP?

Our Contributions to the Literature

1. We employ panel data methods that account for (i) cross-country heterogeneity, (ii) dynamics and (iii) cross-sectional dependences. Existing literature that we survey below do not allow for all these important features of the data.
2. By using a cross-sectionally augmented autoregressive distributed lag (CS-ARDL) modelling approach based on Chudik and Pesaran (2013), we distinguish between long-run and short-run effects and deal with the endogeneity of inflation and debt for estimation of long-run effects.
2. Moreover, we outline a new direct approach to estimating long-run effects that is based on a cross section augmented distributed lag representation of the data (CS-DL approach). CS-DL approach has a number of advantages over the CS-ARDL approach, and provide further robustness evidence on the debt (inflation)-growth relationship.

4. We study the implications of heterogeneity in shaping the debt-inflation-growth relationship across countries and stress the importance of accounting for cross-country interconnectedness arising from global factors.
5. We jointly model inflation, debt, and growth for a panel of advanced and emerging market economies over the period 1966–2010 and investigate whether a build-up of public debt slows down the economy in the *long run*.
6. We also examine possible nonlinearity between debt/GDP ratio and output growth, but distinguish between rising and falling debt once the threshold is breached.

Roadmap

- Estimation of long-run relationships in economics
  - Illustrative example, ARDL and DL approaches and their merits
- CS-DL mean group and pooled estimators
- Monte Carlo evidence
- Debt, inflation and growth: Literature review
- Data used in our empirical application
- Empirical findings
- Conclusion

Estimation of Long-Run (LR) Relationships in Economics

- Estimating LR relationships is of great importance in economics.
- LR relationships are less controversial than short-run relationships (which are model specific and subject to identification problems)
  - many LR relationships in economics are free of particular model assumptions (UIP, PPP and Fisher inflation parity can be obtained by arbitrage within and across markets)
  - other LR relations (e.g. those between macro economic aggregates like consumption and income, output and investment, technological progress and real wages) are less grounded in arbitrage, but still form a major part of what is generally agreed in empirical macro modelling.
- We focus on the estimation of long-run relations without restricting the short-run dynamics.

Example (From VAR to ARDL)

Let $\mathbf{z}_t = (y_t, x_t)'$ be jointly determined in a VAR(1) model,

$$\mathbf{z}_t = \mathbf{\Phi}\mathbf{z}_{t-1} + \mathbf{e}_t, \tag{1}$$

Denoting $\text{cov}(e_{yt}, e_{xt})$ by $\omega Var(e_{xt})$, we can write

$$e_{yt} = E(e_{yt} | e_{xt}) + u_t = \omega e_{xt} + u_t, \tag{2}$$

where by construction $E(u_t | e_{xt}) = 0$. Substituting (2) for e_{yt}, the equation for the dependent variable y_t in (1) is

$$y_t = \phi_{11} y_{t-1} + \phi_{12} x_{t-1} + \omega e_{xt} + u_t. \tag{3}$$

Using the equation for x_t in (1), we obtain
$e_{xt} = x_t - \phi_{21} y_{t-1} - \phi_{22} x_{t-1}$, and substituting this back in (3) yields,

$$y_t = \varphi y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + u_t, \tag{4}$$

where $\varphi = \phi_{11} - \omega\phi_{21}$, $\beta_0 = \omega$, $\beta_1 = \phi_{12} - \omega\phi_{22}$.

- The level coefficient in this example is defined by the ratio

$$\theta = \frac{\beta_0 + \beta_1}{1 - \varphi}.$$

- $(1, -\theta)'$ is cointegrating vector if $\mathbf{z}_t$ is $I(1)$.
- θ can be motivated also in the stationary case by a counterfactual exercise.
- θ (and short-run parameters) can be directly estimated from ARDL model (4) *regardless* of whether x_t is exogenous or not.

Two Approaches to Estimating LR Coefficients in Panels

Consider the following panel extension:

$$y_{it} = \sum_{\ell=1}^{p_{yi}} \varphi_{i\ell} y_{i,t-\ell} + \sum_{\ell=0}^{p_{xi}} \boldsymbol{\beta}'_{i\ell} \mathbf{x}_{i,t-\ell} + u_{it}, \tag{5}$$

$$u_{it} = \gamma_i' \mathbf{f}_t + \varepsilon_{it}, \tag{6}$$

for $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, where $\mathbf{f}_t$ is an $m \times 1$ vector of unobserved common factors, and p_{yi} and p_{xi} are the lag orders chosen to be sufficiently long so that u_{it} is a serially uncorrelated process across all i. The vector of long-run coefficients is then given by

$$\boldsymbol{\theta}_i = \frac{\sum_{\ell=0}^{p_{xi}} \boldsymbol{\beta}_{i\ell}}{1 - \sum_{\ell=1}^{p_{yi}} \varphi_{i\ell}}. \tag{7}$$

CS-ARDL approach: Estimation of $\boldsymbol{\theta} = E(\boldsymbol{\theta}_i)$ is based on estimates of short run coefficients in ARDL representation (5). The effects of factors are dealt with by appropriate augmentation of the ARDL model by CS averages (Pesaran, 2006, Chudik and Pesaran, 2013).

- **Pros**: Estimates of $\boldsymbol{\theta}$ are valid regardless of whether variables are endogenous, exogenous, $I(0)$, or $I(1)$. Techniques that take care of effects of factors are now available (Song 2013, and Chudik and Pesaran, 2013)
- **Cons**: Sample uncertainty could be large when $\sum_{\ell=1}^{p_{yi}} \varphi_{i\ell}$ is close to one. This method performs well only when T is sufficiently large. Correct specification of lags is important. LS estimates are affected by $O\left(T^{-1}\right)$ small sample bias.

CS-DL approach: Estimation of $\boldsymbol{\theta} = E\left(\boldsymbol{\theta}_i\right)$ is based on a DL representation

$$y_{it} = \boldsymbol{\theta}_i \mathbf{x}_{it} + \boldsymbol{\alpha}_i'\left(L\right)\Delta\mathbf{x}_{it} + \tilde{u}_{it}, \tag{8}$$

where $\tilde{u}_{it} = \varphi\left(L\right)^{-1} u_{it}$, $\varphi_i\left(L\right) = 1 - \sum_{\ell=1}^{p_{yi}} \varphi_{i\ell} L^{\ell}$, $\boldsymbol{\theta}_i = \boldsymbol{\delta}_i\left(1\right)$, $\boldsymbol{\delta}_i\left(L\right) = \varphi_i^{-1}\left(L\right)\boldsymbol{\beta}_i\left(L\right) = \sum_{\ell=0}^{\infty} \boldsymbol{\delta}_{i\ell} L^{\ell}$, $\boldsymbol{\beta}_i\left(L\right) = \sum_{\ell=0}^{p_{xi}} \boldsymbol{\beta}_{i\ell} L^{\ell}$, and $\boldsymbol{\alpha}_i\left(L\right) = \sum_{\ell=0}^{\infty} \boldsymbol{\alpha}_{i\ell} L^{\ell}$ with $\boldsymbol{\alpha}_{i\ell} = \sum_{s=\ell+1}^{\infty} \boldsymbol{\delta}_s$, for $\ell = 0, 1, 2, \ldots$.
Appropriate augmentation by CS averages can again deal with the effects of factors in $\tilde{u}_{it}$.

- **Pros**: Superior small sample performance when T is moderate vs. ARDL method for a range of experiments. Robustness to breaks in errors. No need to specify lag orders p_{yi} and p_{xi}.
- **Cons**: No feedback effects from y_{it} to regressors allowed (consistency), performance deteriorates when eigenvalues of $\varphi_i\left(L\right)$ are close to unit circle.

CS-DL Mean Group and Pooled Estimators

- We continue to work with the panel ARDL model (5)-(6) but assume $p_{yi} = 1$ and $p_{xi} = 0$ for simplicity and without loss of generality.
- The DL representation is thus given by

$$
\begin{aligned}
y_{it} &= (1 - \varphi_i L)^{-1} \boldsymbol{\beta}_i' \mathbf{x}_{it} + (1 - \varphi_i L)^{-1} \boldsymbol{\gamma}_i' \mathbf{f}_t + (1 - \varphi_i L)^{-1} \varepsilon_{it} \\
&= \boldsymbol{\theta}_i \mathbf{x}_{it} - \boldsymbol{\alpha}_i' (L) \Delta \mathbf{x}_{it} + \boldsymbol{\gamma}_i' \tilde{\mathbf{f}}_{it} + \tilde{\varepsilon}_{it}, \text{ for } i = 1, 2, \ldots, N,
\end{aligned}
$$

where $\Delta \mathbf{x}_{it} = \mathbf{x}_{it} - \mathbf{x}_{i,t-1}$, $\boldsymbol{\alpha}_i (L) = \sum_{\ell=0}^{\infty} \boldsymbol{\alpha}_{i\ell} L^{\ell}$, $\alpha_{i\ell} = \varphi_i^{\ell+1} \sum_{\ell=0}^{\infty} \varphi_i^{\ell} \boldsymbol{\beta}_i$, for $\ell = 0, 1, 2, \ldots$, $\tilde{\mathbf{f}}_{it} = (1 - \varphi_i L)^{-1} \mathbf{f}_t$ and $\tilde{\varepsilon}_{it} = (1 - \varphi_i L)^{-1} \varepsilon_{it}$.

- The idea of dealing with the lagged dependent variable problem is to augment the individual regressions by differences of unit specific regressors $\mathbf{x}_{it}$ and their lags, in addition to the augmentation by the cross section averages that take care of the effects of unobserved common factors.

Some Notations

Define the cross section averages $\bar{\mathbf{z}}_{wt} = (\bar{y}_{wt}, \bar{\mathbf{x}}'_{wt})' = \sum_{i=1}^{N} w_i \mathbf{z}_{it}$, where the weights $\{w_i\}$ satisfy the usual 'granularity' conditions. Define also the following data vectors and matrices,

$\mathbf{y}_i = (y_{i,p+1}, y_{i,p+2}, ..., y_{i,T})'$, $\mathbf{X}_i = (\mathbf{x}_{i,p+1}, \mathbf{x}_{i,p+2}, ..., \mathbf{x}_{i,T})'$,
$\bar{\mathbf{Z}}_w = (\bar{\mathbf{z}}_{w,p+1}, \bar{\mathbf{z}}_{w,p+2}, ..., \bar{\mathbf{z}}_{w,T})'$,

$$\underset{(T-p)\times(p+1)k}{\Delta \mathbf{X}_{ip}} = \begin{pmatrix} \Delta \mathbf{x}'_{i,p+1} & \Delta \mathbf{x}'_{i,p} & \cdots & \Delta \mathbf{x}'_{i1} \\ \vdots & \vdots & & \vdots \\ \Delta \mathbf{x}'_{iT} & \Delta \mathbf{x}'_{i,T-1} & \cdots & \Delta \mathbf{x}'_{i,T-p} \end{pmatrix},$$

$\mathbf{Q}_{wi} = \left(\bar{\mathbf{Z}}_w, \Delta\bar{\mathbf{X}}_{wp}, \Delta \mathbf{X}_{ip}\right)$, $\Delta\bar{\mathbf{X}}_{wp} = \sum_{i=1}^{N} w_i \Delta \mathbf{X}_{ip}$, and

$$\mathbf{M}_{qi} = \mathbf{I}_{T-p} - \mathbf{Q}_{wi}\left(\mathbf{Q}'_{wi}\mathbf{Q}_{wi}\right)^{+}\mathbf{Q}'_{wi},$$

for $i = 1, 2, ..., N$, where $p = p(T)$ is non-decreasing such that $0 \leq p < T$.

- All of these vectors and matrices depend on the chosen truncation lag p, but in most instances we suppress the subscript p to simplify notations.

- The CS-DL mean group estimator of the mean long run coefficients is given by

$$\widehat{\boldsymbol{\theta}}_{MG} = \frac{1}{N}\sum_{i=1}^{N}\widehat{\boldsymbol{\theta}}_i, \tag{9}$$

  where

$$\widehat{\boldsymbol{\theta}}_i = \left(\mathbf{X}_i'\mathbf{M}_{qi}\mathbf{X}_i'\right)^{-1}\mathbf{X}_i'\mathbf{M}_{qi}\mathbf{y}_i.$$

- The CS-DL pooled estimator of the mean long run coefficients is

$$\widehat{\boldsymbol{\theta}}_P = \left(\sum_{i=1}^{N} w_i\mathbf{X}_i'\mathbf{M}_{qi}\mathbf{X}_i\right)^{-1}\sum_{i=1}^{N} w_i\mathbf{X}_i'\mathbf{M}_{qi}\mathbf{y}_i. \tag{10}$$

- Estimators $\widehat{\boldsymbol{\theta}}_{MG}$ and $\widehat{\boldsymbol{\theta}}_P$ differ from the mean group and pooled CCE estimator developed in Pesaran (2006) in the inclusion of the first differences of regressors $\mathbf{x}_{it}$ and their lags.

Overview of Main Theoretical Findings

- We establish asymptotic normality of $\widehat{\boldsymbol{\theta}}_{MG}$ and $\widehat{\boldsymbol{\theta}}_{P}$ under the coefficient heterogeneity and asymptotics $(N, T, p) \xrightarrow{j} \infty$ such that $\sqrt{N} p \rho^{p} \to 0$ for any constant $0 < \rho < 1$ and $p^3/T \to \varkappa$, $0 < \varkappa < \infty$.
- p increases in T, but not too fast (sufficient degrees of freedom), and not too slow (truncation error is negligible). We set $p = \left[T^{1/3} \right]$.
- We also propose robust estimates of standard errors (this is important for valid inference).

- The CS-DL estimator of the long-run coefficients are robust to a number of features:
  - Unknown serial correlation in covariance stationary idiosyncratic errors, ε_{it}.
  - Arbitrary spatial/weak cross sectional dependence of ε_{it} (row norm of $E\left(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\right)$ is assumed bounded in N).
  - Arbitrary fixed number of factors (subject to certain conditions).
  - Regressors correlated with factors as in $\mathbf{x}_{it} = \boldsymbol{\Gamma}_i'\mathbf{f}_t + \mathbf{v}_{it}$, where $\mathbf{v}_{it}$ allows to follow arbitrary covariance stationary process.

MC Experiments I

- **Homogeneity of coefficients**: Case 1: LR homogeneous, SR heterogeneous, Case 2: both SR and LR homogeneous. We find no visible size distortions of CS-DL estimators even when $T = 30$ (size for $T = 30$ in the range 4.6% to 7.2%).
- **Different lag orders** in the ARDL data generating process: we set $p_y = 1$ and $p_x = 0$. The performance of CS-DL estimators remains good.
- **Sensitivity to $\varphi_{\max}$**: We consider $\varphi_{\max} = 0.8$ and 0.9 as opposed to 0.6 in the baseline. The performance of CS-DL deteriorates significantly as $\varphi_{\max}$ gets closer to 1.
- **Robustness to number of factors:** We also consider a rank deficient case with $m = 3$ factors and find very good performance of CS-DL estimators. CS-ARDL estimators, on the other hand, are not consistent in the rank deficient case with serially correlated factors.

MC Experiments II

- **Unit roots in factors or regressor specific components**. The size and overall performance remains good.
- **Serial correlation in idiosyncratic errors and break in errors**. CS-ARDL estimators are no longer consistent. However, the performance of CS-DL continues to be satisfactory even in the case of breaks.
- **Consequences of feedback effects** from y_{it} to x_{it}. CS-DL approach is no longer consistent, whereas CS-ARDL approach remains valid. However, we observe better performance of CS-DL approach even in the presence of feedback effects when $T < 100$.
- We also consider whether imposing CS-DL pooled estimates of long-run in ARDL models can help to improve the estimation of short-run parameters. We find that it can substantially improve the estimation of short-run coefficients - about 80-90% reduction of the difference between the infeasible estimator (with the knowledge of LR coefficient imposed) and unrestricted CS-ARDL estimator.

Debt, Inflation and Growth

Literature review I

- Economic theory provides mixed predictions on the effects of inflation on economic growth (depending on how money is introduced into the model)– Tobin (1965); Sidrauski (1967); Dornbusch and Frenkel (1973); Stockman (1981); Gomme (1993); Ireland (1994).
- Similar theoretical ambiguities prevail regarding the relationship between public debt and growth.
  - On the one hand, profligate debt-generating fiscal policy (and high public debt) can have a negative impact on long-term economic growth by crowding out private investment– Elmendorf and Mankiw (1999); Cochrane (2011).
  - On the other hand, hysteresis arising from recessions can lead to a situation in which expansionary fiscal policies may have positive effect on growth– DeLong and Summers (2012).
- Hence, whether high levels of public debt can have a negative effect on long-run growth is an empirical question– Reinhart and Rogoff (2010).

Literature Review II

- The inflation-growth relationship is not robust, due to the sample selection bias, temporal aggregation, and endogeneity– Ericsson et al. (2001).
- Possible short-run and long-run effects of inflation and public debt on growth need not be the same and must be distinguished empirically.
- Recent literature also assessed whether Reinhart and Rogoff's results are robust to selecting non-arbitrary debt brackets; control variables in a multivariate regression framework; reverse causality; cross-country heterogeneity; changes in country coverage; data frequency; and econometric specification– Kumar and Woo (2010); Checherita-Westphal and Rother (2012); Eberhardt and Presbitero (2012); Panizza and Presbitero (2013).

Literature Review III

- Reinhart and Rogoff (2010) argue for a non-linear relationship between debt and growth. Krugman (1988) argues for nonlinearities and threshold effects arising from the presence of external debt overhang.
- Eberhardt and Presbitero (2012) model a level relationship between output, debt/GDP and investment/GDP ratios in a heterogenous panel data model. However, their analysis suffer from a number of shortcomings: assume cointegration with out testing, difficult to interpret due to the inclusion of investment-output ratio, and insufficient dynamics and hence the possibility of the endogeneity problem.

Data I

- The CPI and real GDP data series are from the IMF *International Financial Statistics* database except for CPI data for Brazil, China and Tunisia which is from the IMF *World Economic Outlook* database and CPI data for UK which is from the Reinhart and Rogoff's *Growth in a Time of Debt* database.
- The gross government debt/GDP data series are from Reinhart and Rogoff (2011) and their most-up-to date *From Financial Crash to Debt Crisis* online database, except for Iran, Morocco, Nigeria, and Syria for which the IMF FAD *Historical Public Debt* database was used instead.
- We focus on gross debt data due to difficulty of collecting net debt data on a consistent basis over time and across countries. Moreover, we use public debt at the general government level for as many countries as possible.

Data II

- Since our analysis allows for slope heterogeneity across countries, we need a sufficient number of time periods to estimate country-specific coefficients. To this end, we include only countries in our sample for which we have at least 30 consecutive annual observations on debt, inflation and GDP.

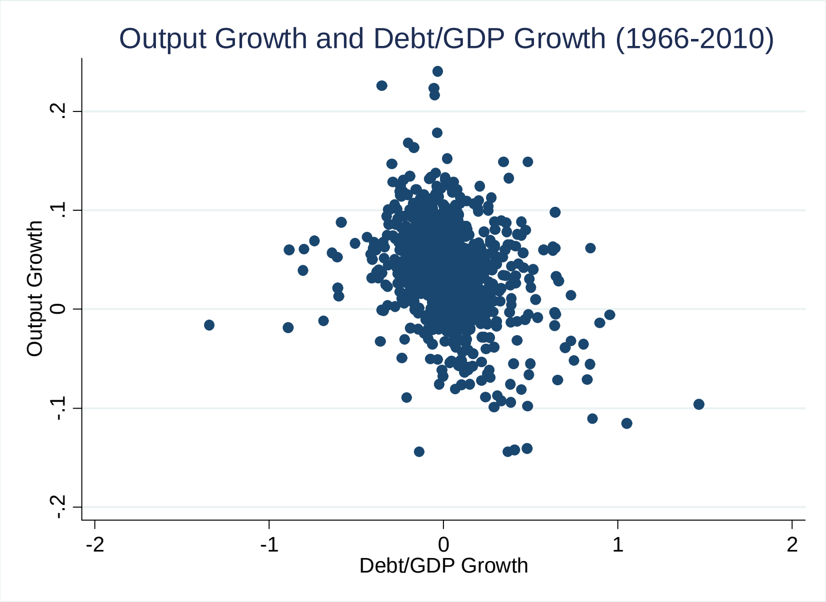
- Subject to this requirement we ended up with 40 countries (covering most regions in the world and include advanced, emerging and developing countries).

- We also set the minimum cross section dimension to 20, since to take account of error cross sectional dependence we need to form cross section averages based on a sufficient number of units. We ended up with an unbalanced panel covering the sample period 1965-2010, with $T_{\text{min}} = 30$, and $N_{\text{min}} = 20$ across all countries and time periods.

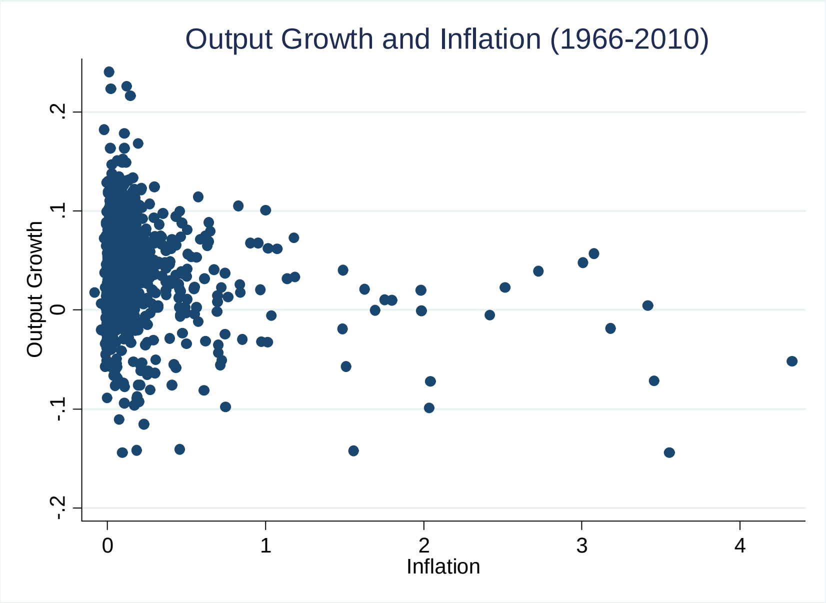
The 40 Countries in Our Sample

Europe	MENA Countries	Asia Pacific	Latin America
Austria	Egypt	Australia	Argentina
Belgium	Iran	China	Brazil
Finland	Morocco	India	Chile
France	Syria	Indonesia	Ecuador
Germany	Tunisia	Japan	Peru
Italy	Turkey	Korea	Venezuela
Netherlands		Malaysia	
Norway	**North America**	New Zealand	**Rest of Africa**
Spain	Canada	Philippines	Nigeria
Sweden	Mexico	Singapore	South Africa
Switzerland	United States	Thailand	
United Kingdom			

Data III


Output Growth and Debt/GDP Growth (1966-2010)
Output Growth
.2
.1
0
-.1
-.2
-2
-1
0
1
2
Debt/GDP Growth

Data IV



Output Growth and Inflation (1966-2010)
Output Growth
.2
.1
0
-.1
-.2
0
1
2
3
4
Inflation

Results I

- First we estimate the following panel ARDL model (without augmentation by CS averages):

$$\Delta y_{it} = c_i + \sum_{l=1}^{p} \varphi_{il} \Delta y_{i,t-l} + \sum_{l=0}^{p} \boldsymbol{\beta}_{il}' \mathbf{x}_{i,t-l} + u_{it},$$

where Δy_{it} is output growth and (a) $x_{it} = \Delta d_{it}$, (b) $x_{it} = \pi_{it}$, or (c) $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})'$. We consider FE and MG estimation methods.

- Next, we estimate CS-ARDL and CS-DL regressions,

$$\Delta y_{it} = c_i + \sum_{l=1}^{p} \varphi_{il} \Delta y_{i,t-l} + \sum_{l=0}^{p} \boldsymbol{\beta}_{il}' \mathbf{x}_{i,t-l} + \sum_{l=0}^{3} \boldsymbol{\psi}_{il}' \bar{\mathbf{z}}_{t-l} + e_{it},$$

and

$$\Delta y_{it} = c_i + \boldsymbol{\theta}_i' \mathbf{x}_{it} + \sum_{l=0}^{p} \boldsymbol{\delta}_{il}' \Delta \mathbf{x}_{i,t-l} + \omega_{iy} \overline{\Delta y}_t + \sum_{l=0}^{3} \boldsymbol{\omega}_{i,xl}' \bar{\mathbf{x}}_{t-l} + e_{it}.$$

Results II

- Our results show that, regardless of the threshold, there are significant and robust negative long-run effects of debt on economic growth.
- By comparison, the evidence of a negative effect of inflation on growth is less strong, although it is often statistically significant in most specifications.

Table 1: FE Estimates based on the ARDL Approach (1966-2010)

	ARDL (1 lag)			ARDL (2 lags)			ARDL (3 lags)		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
$\widehat{\theta}_{\Delta d}$	-0.075***		-0.069***	-0.061***		-0.054***	-0.055***		-0.044***
	(0.009)		(0.008)	(0.010)		(0.009)	(0.016)		(0.013)
$\widehat{\theta}_{\pi}$		-0.025***	-0.025***		-0.025***	-0.026***		-0.025***	-0.024***
		(0.007)	(0.004)		(0.007)	(0.006)		(0.008)	(0.006)
$\widehat{\lambda}$	-0.854***	-0.790***	-0.876***	-0.839***	-0.771***	-0.861***	-0.768***	-0.723***	-0.771***
	(0.052)	(0.064)	(0.051)	(0.045)	(0.051)	(0.048)	(0.045)	(0.039)	(0.049)
CD TS	24.52	34.72	26.35	23.20	34.90	24.96	21.85	32.68	23.31
$N \times T$	1642	1725	1642	1602	1685	1602	1562	1645	1562

Notes: The ARDL specification is given by:

$$\Delta y_{it} = c_i + \sum_{l=1}^{p} \varphi_{il} \Delta y_{i,t-l} + \sum_{l=0}^{p} \boldsymbol{\beta}_{il}' \mathbf{x}_{i,t-l} + u_{it},$$

where Δy_{it} is output growth and (a) $x_{it} = \Delta d_{it}$, (b) $x_{it} = \pi_{it}$, (c) $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})'$. $\lambda_i = 1 - \sum_{l=1}^{p} \varphi_{il}$ and $\theta_i = \lambda_i^{-1} \sum_{l=0}^{p} \boldsymbol{\beta}_{il}$. Symbols ***, **, and * denote significance at 1%, 5%, and at 10% respectively.

Table 2: MG Estimates based on the ARDL Approach (1966-2010)

	ARDL (1 lag)			ARDL (2 lags)			ARDL (3 lags)		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
$\hat{\theta}_{\Delta d}$	-0.070***	–	-0.070***	-0.061***	–	-0.076***	-0.066***	–	-0.083***
	(0.015)		(0.012)	(0.014)		(0.013)	(0.016)		(0.014)
$\hat{\theta}_{\pi}$	–	-0.104***	-0.038*	–	-0.054**	0.021	–	-0.091***	0.040
		(0.021)	(0.023)		(0.024)	(0.030)		(0.032)	(0.040)
$\hat{\lambda}$	-0.791***	-0.764***	-0.811***	-0.836***	-0.742***	-0.809***	-0.769***	-0.687***	-0.761***
	(0.028)	(0.037)	(0.030)	(0.039)	(0.044)	(0.047)	(0.043)	(0.041)	(0.053)
CD TS	19.15	33.62	21.39	16.99	31.21	16.63	16.42	30.39	15.98
$N \times T$	1642	1725	1642	1602	1685	1602	1562	1645	1562

Notes: The ARDL specification is given by:

$$\Delta y_{it} = c_i + \sum_{l=1}^{p} \varphi_{il} \Delta y_{i,t-l} + \sum_{l=0}^{p} \boldsymbol{\beta}_{il}' \mathbf{x}_{i,t-l} + u_{it},$$

where Δy_{it} is output growth and (a) $x_{it} = \Delta d_{it}$, (b) $x_{it} = \pi_{it}$, (c) $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})'$. $\lambda_i = 1 - \sum_{l=1}^{p} \varphi_{il}$ and $\theta_i = \lambda_i^{-1} \sum_{l=0}^{p} \boldsymbol{\beta}_{il}$. Symbols ***, **, and * denote significance at 1%, 5%, and at 10% respectively.

Table 3: MG Estimates based on the CS-ARDL Approach (1966-2010)

	CS-ARDL (1 lag)			CS-ARDL (2 lags)			CS-ARDL (3 lags)		
	(a)	*(b)*	*(c)*	*(a)*	*(b)*	*(c)*	*(a)*	*(b)*	*(c)*
$\widehat{\theta}_{\Delta d}$	-0.087***	–	-0.087***	-0.090***	–	-0.079***	-0.096***	–	-0.120***
	(0.013)		(0.016)	(0.013)		(0.022)	(0.016)		(0.040)
$\widehat{\theta}_{\pi}$	–	-0.083**	-0.164***	–	-0.071**	-0.110***	–	-0.065	-0.080
		(0.034)	(0.038)		(0.031)	(0.035)		(0.041)	(0.059)
$\widehat{\lambda}$	-0.889***	-0.790***	-0.952***	-0.967***	-0.817***	-1.058***	-0.920***	-0.792***	-1.210***
	(0.031)	(0.041)	(0.039)	(0.042)	(0.053)	(0.053)	(0.047)	(0.058)	(0.201)
CD TS	-0.94	-0.30	0.55	-0.43	0.02	-0.11	-0.21	0.05	-0.56
$N \times T$	1599	1657	1599	1581	1652	1581	1562	1645	1562

Notes: The cross-sectionally augmented ARDL (CS-ARDL) specification is given by:

$$\Delta y_{it} = c_i + \sum_{l=1}^{p} \varphi_{il} \Delta y_{i,t-l} + \sum_{l=0}^{p} \boldsymbol{\beta}_{il}' \mathbf{x}_{i,t-l} + \sum_{l=0}^{3} \boldsymbol{\psi}_{il}' \bar{\mathbf{z}}_{t-l} + e_{it},$$

where $\bar{\mathbf{z}} = \left(\overline{\Delta y}_t, \bar{\mathbf{x}}_t'\right)'$ Δy_{it} is output growth and (a) $x_{it} = \Delta d_{it}$, (b) $x_{it} = \pi_{it}$, (c) $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})'$. $\lambda_i = 1 - \sum_{l=1}^{p} \varphi_{il}$ and $\theta_i = \lambda_i^{-1} \sum_{l=0}^{p} \boldsymbol{\beta}_{il}$. Symbols ***, **, and * denote significance at 1%, 5%, and at 10% respectively.

Table 4: MG Estimates based on the CS-DL Approach (1966-2010)

	CS-DL (1 lag)			CS-DL (2 lags)			CS-DL (3 lags)		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
$\widehat{\theta}_{\Delta d}$	-0.084***	–	-0.087***	-0.078***	–	-0.084***	-0.068***	–	-0.082***
	(0.013)		(0.014)	(0.014)		(0.017)	(0.014)		(0.020)
$\widehat{\theta}_{\pi}$	–	-0.066***	-0.089***	–	-0.072***	-0.086**	–	-0.072**	-0.086**
		(0.021)	(0.026)		(0.024)	(0.037)		(0.030)	(0.040)
CD TS	-1.54	-0.21	1.16	-1.23	0.17	0.73	-1.09	-0.46	0.63
$N \times T$	1601	1661	1601	1586	1661	1586	1571	1660	1571

Notes: The cross-sectionally augmented distributed lag (CS-DL) specification:

$$\Delta y_{it} = c_i + \boldsymbol{\theta}_i' \mathbf{x}_{it} + \sum_{l=0}^{p} \delta_{il}' \Delta \mathbf{x}_{i,t-l} + \omega_{iy} \overline{\Delta y}_t + \sum_{l=0}^{3} \boldsymbol{\omega}_{i,xl}' \bar{\mathbf{x}}_{t-l} + e_{it},$$

where Δy_{it} is output growth and (a) $x_{it} = \Delta d_{it}$, (b) $x_{it} = \pi_{it}$, (c) $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})'$. Symbols ***, **, and * denote significance at 1%, 5%, and at 10% respectively.

Threshold Effects

- Potentially, the relationship between debt (inflation) and growth could be non-linear (involve threshold effects).
  - See Roubini and Sala-i-Martin (1992); Ghosh and Phillips (1998); Khan and Senhadji (2001) for a discussion of non-linearities and threshold effects for inflation-growth nexus. Bruno and Easterly (1998) also argue that growth could be negatively associated with high inflation crises.
  - See Reinhart and Rogoff (2010); Krugman (1988); Ghosh et al. (2013) for a discussion of non-linearities and threshold effects in the relationship between growth and public debt, and the presence of "external debt overhang".
- We study whether there is a common threshold for government debt ratios above which long-term growth rates are adversely affected (especially if the country is on an upward debt trajectory).
- We particularly focus on debt *trajectory* beyond certain debt-level thresholds as to the best of our knowledge no such econometric analysis has been carried out in the past.

Results III

We begin by estimating separate specifications for different threshold effects (denoted by τ) but assume that the threshold effect is homogenous across countries for a given debt bracket:

$$(i) \quad \Delta y_{it} = c_{\tau} + \gamma_{\tau} I_{it}(\tau) + e_{it},$$

Relaxing the slope homogeneity assumption we obtain MG estimates using the specification:

$$(ii) \quad \Delta y_{it} = c_{i\tau} + \gamma_{i\tau} I_{it}(\tau) + e_{it},$$

where $I_{it}(\tau) = I(d_{it} \geq \log(\tau))$ is an indicator function, τ is a common debt threshold, and as before Δy_{it} is output growth.

We also allow for cross-sectional dependence

$$
\begin{aligned}
(iii) \quad \Delta y_{it} \quad &= \quad c_{i\tau} + \gamma_{i\tau} I_{it}(\tau) + \boldsymbol{\theta}_i^{'} \mathbf{x}_{it} \\
&\quad + \sum_{\ell=0}^{2} \delta_{i\ell}^{'} \Delta \mathbf{x}_{i,t-\ell} + \omega_{iy} \overline{\Delta y}_t + \sum_{\ell=0}^{3} \boldsymbol{\omega}_{i,x\ell}^{'} \bar{\mathbf{x}}_{t-\ell} + e_{it},
\end{aligned}
$$

and consider the inclusion of interactive effects where the threshold effects kick in only in the case of rising debt-GDP ratios

$$
\begin{aligned}
(iv) \quad \Delta y_{it} \quad &= \quad c_{i\tau} + \gamma_{i\tau}^{+} \left[I_{it}(\tau) \times \max\left(0, \Delta d_{it}\right) \right] + \boldsymbol{\theta}_{i,\tau}^{'} \mathbf{x}_{it} \\
&\quad + \sum_{\ell=0}^{2} \delta_{i\ell,\tau}^{'} \Delta \mathbf{x}_{i,t-\ell} + \omega_{iy,\tau} \overline{\Delta y}_t + \sum_{\ell=0}^{3} \boldsymbol{\omega}_{i,x\ell,\tau}^{'} \bar{\mathbf{x}}_{t-\ell} + e_{it},
\end{aligned}
$$

where $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})'$.

Results IV

- Our results show that there is no simple common threshold for the *level* of government debt above which long-term growth is adversely affected.

- The results also suggest that if debt-GDP level is increased and this increase is permanent, then it will have negative effects on the economic growth in the long run. On the other hand, if the debt is increased (for instance to help smooth out business cycle fluctuations) and this increase is temporary, then there are no long-run negative effects on output growth.

Table 5: Threshold Effects (1966-2010)

τ	30	40	50	60	70	80	90
(i) Pooled OLS Estimates with $I_{it}(\tau)$, where $I_{it}(\tau) = I(d_{it} \geq \log(\tau))$							
$\widehat{\gamma}_\tau$	-0.008***	-0.009***	-0.009***	-0.009***	-0.009***	-0.009***	-0.011***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)
$\hat{c}_\tau$	0.043***	0.042***	0.041***	0.040***	0.039***	0.039***	0.039***
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
N	40	40	40	40	40	40	40
$N \times T$	1696	1696	1696	1696	1696	1696	1696
(ii) Mean Group Estimates with $I_{it}(\tau)$							
$\widehat{\gamma}_\tau$	-0.008**	-0.010***	-0.012***	-0.011***	-0.016***	-0.020***	-0.021***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)
$\hat{c}_\tau$	0.045***	0.046***	0.043***	0.041***	0.041***	0.044***	0.048***
	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)
N	32	36	31	31	28	19	14
$N \times T$	1353	1531	1322	1332	1203	810	589

Notes: The estimates are based on the following specifications:

$$(i)\ \Delta y_{it} = c_\tau + \gamma_\tau I_{it}(\tau) + e_{it},$$
$$(ii)\ \Delta y_{it} = c_{i\tau} + \gamma_{i\tau} I_{it}(\tau) + e_{it},$$

where $I_{it}(\tau) = I(d_{it} \geq \log(\tau))$, y_{it} is the log of real GDP, $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})'$, d_{it} is the log of the debt GDP ratio, and π_{it} is the inflation rate. We report heteroscedasticity-robust standard errors for specification (i). Symbols ***, **, and * denote significance at 1%, 5%, and at 10% respectively.

Table 5(Cont.): Threshold Effects (1966-2010)

τ	30	40	50	60	70	80	90
(iii) CS-DL Mean Group Estimates with $I_{it}(\tau)$							
$\widehat{\gamma}_{\tau}$	-0.006	-0.004	-0.008	-0.005	-0.009	-0.001	-0.006
	(0.009)	(0.005)	(0.009)	(0.006)	(0.006)	(0.009)	(0.007)
$\widehat{\theta}_{\tau,\Delta d}$	-0.071***	-0.087***	-0.076***	-0.063**	-0.076***	-0.089***	-0.109***
	(0.024)	(0.022)	(0.025)	(0.026)	(0.025)	(0.031)	(0.037)
$\widehat{\theta}_{\tau,\pi}$	-0.095*	-0.062	-0.090*	-0.079	-0.161***	-0.138**	-0.142
	(0.050)	(0.045)	(0.052)	(0.049)	(0.053)	(0.061)	(0.110)
N	32	35	31	31	28	18	14
$N \times T$	1251	1377	1226	1236	1115	710	547

Notes: The estimates are based on the specification (*iii*):

$$\Delta y_{it} = c_{i\tau} + \gamma_{i\tau} I_{it}(\tau) + \boldsymbol{\theta}_i' \mathbf{x}_{it} + \sum_{\ell=0}^{2} \delta_{i\ell}' \Delta \mathbf{x}_{i,t-\ell} + \omega_{iy} \overline{\Delta y}_t + \sum_{\ell=0}^{3} \boldsymbol{\omega}_{i,x\ell}' \bar{\mathbf{x}}_{t-\ell} + e_{it},$$

where $I_{it}(\tau) = I(d_{it} \geq \log(\tau))$, y_{it} is the log of real GDP, $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})'$, d_{it} is the log of the debt GDP ratio, and π_{it} is the inflation rate. Symbols ***, **, and * denote significance at 1%, 5%, and at 10% respectively.

Table 5(Cont.): Threshold Effects (1966-2010)

τ	30%	40%	50%	60%	70%	80%	90%
(iv) CS-DL Mean Group Estimates with $I_{it}(\tau)$ and $I_{it}(\tau) \times \max(0, \Delta d_{it})$							
$\widehat{\gamma}_{\tau}$	0.002	0.001	-0.006	-0.005	-0.018	-0.009	-0.001
	(0.005)	(0.005)	(0.007)	(0.006)	(0.011)	(0.015)	(0.018)
$\widehat{\gamma}_{\tau}^{+}$	-0.005	0.018	-0.028	-0.116***	-0.127	-0.192**	-0.140**
	(0.025)	(0.024)	(0.038)	(0.045)	(0.080)	(0.094)	(0.062)
$\widehat{\theta}_{\tau,\Delta d}$	-0.085***	-0.100***	-0.079***	-0.050*	-0.064**	-0.088***	-0.100***
	(0.031)	(0.025)	(0.028)	(0.027)	(0.028)	(0.034)	(0.038)
$\widehat{\theta}_{\tau,\pi}$	-0.119**	-0.073	-0.099**	-0.085**	-0.155***	-0.125*	-0.118
	(0.047)	(0.047)	(0.049)	(0.039)	(0.057)	(0.064)	(0.103)
N	30	33	31	31	25	18	14
$N \times T$	1184	1310	1226	1236	999	710	547

Notes: The estimates are based on specification (iv):

$$\Delta y_{it} = c_{i\tau} + \gamma_{i\tau} I_{it}(\tau) + \gamma_{i\tau}^{+}\left[I_{it}(\tau) \times \max(0, \Delta d_{it})\right] + \boldsymbol{\theta}_{i,\tau}' \mathbf{x}_{it} + \sum_{\ell=0}^{2} \boldsymbol{\delta}_{i\ell,\tau}' \Delta \mathbf{x}_{i,t-\ell} + \omega_{iy,\tau} \overline{\Delta y}_{t} + \sum_{\ell=0}^{3} \boldsymbol{\omega}_{i,x\ell,\tau}' \bar{\mathbf{x}}_{t-\ell} + e_{it},$$

where $I_{it}(\tau) = I(d_{it} \geq \log(\tau))$, y_{it} is the log of real GDP, $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})'$, d_{it} is the log of the debt GDP ratio, and π_{it} is the inflation rate. Symbols ***, **, and * denote significance at 1%, 5%, and at 10% respectively.

Table 5(Cont.): Threshold Effects (1966-2010)

τ	30%	40%	50%	60%	70%	80%	90%
(v) CS-DL Mean Group Estimates with $I_{it}(\tau) \times \max(0, \Delta d_{it})$							
$\widehat{\gamma}_{\tau}^{+}$	-0.001	-0.001	-0.060	-0.113***	-0.158***	-0.171***	-0.159***
	(0.024)	(0.024)	(0.040)	(0.044)	(0.057)	(0.052)	(0.046)
$\widehat{\theta}_{\tau,\Delta d}$	-0.090***	-0.100***	-0.069***	-0.056**	-0.070***	-0.066**	-0.080**
	(0.025)	(0.024)	(0.025)	(0.024)	(0.021)	(0.028)	(0.035)
$\widehat{\theta}_{\tau,\pi}$	-0.087**	-0.083**	-0.085*	-0.096**	-0.135***	-0.061	-0.031
	(0.037)	(0.040)	(0.045)	(0.042)	(0.049)	(0.058)	(0.080)
N	38	36	32	31	28	18	14
$N \times T$	1487	1414	1263	1236	1115	710	547

Notes: The estimates are based on specification (v):

$$\Delta y_{it} = c_{i\tau} + \gamma_{i\tau}^{+}\left[I_{it}(\tau) \times \max(0, \Delta d_{it})\right] + \boldsymbol{\theta}_{i,\tau}^{\prime}\mathbf{x}_{it} + \sum_{\ell=0}^{2} \boldsymbol{\delta}_{i\ell,\tau}^{\prime}\Delta\mathbf{x}_{i,t-\ell} + \omega_{iy,\tau}\overline{\Delta y}_{t} + \sum_{\ell=0}^{3} \boldsymbol{\omega}_{i,x\ell,\tau}^{\prime}\bar{\mathbf{x}}_{t-\ell} + e_{it},$$

where $I_{it}(\tau) = I(d_{it} \geq \log(\tau))$, y_{it} is the log of real GDP, $\mathbf{x}_{it} = (\Delta d_{it}, \pi_{it})^{\prime}$, d_{it} is the log of the debt GDP ratio, and π_{it} is the inflation rate. Symbols ***, **, and * denote significance at 1%, 5%, and at 10% respectively.

Results V

- The CS-DL approach is robust to the possibility of unit roots in the regressors and/or factors.
- We therefore investigate the long-run effects of the log-level of debt/GDP ratio (d_{it}) and inflation (π_{it}) on the log-level of output and well as that of the log-level of debt/GDP ratio (d_{it}) and log-level of prices ($p_{it} = \log(CPI_{it})$) on the log-level of output.
- We estimate:

$$y_{it} = c_i + \boldsymbol{\theta}_i^{'} \mathbf{x}_{it} + \sum_{l=0}^{p} \delta_{il}^{'} \Delta \mathbf{x}_{i,t-l} + \omega_{iy} \bar{y}_t + \sum_{l=0}^{3} \boldsymbol{\omega}_{i,xl}^{'} \bar{\mathbf{x}}_{t-l} + e_{it},$$

  where y_{it} is the log-level of output, and (a) $\mathbf{x}_{it} = (d_{it}, \pi_{it})'$ or (b) $\mathbf{x}_{it} = (d_{it}, p_{it})'$.
- Our results show that an increase in the debt/GDP ratio has a negative long-run effect on the level of output.

Table 6: MG Estimates based on the CS-DL Approach (1965-2010)

	CS-DL (1 lag)		CS-DL (2 lags)		CS-DL (3 lags)	
	(a)	(b)	(a)	(b)	(a)	(b)
$\widehat{\theta}_d$	-0.068***	-0.075***	-0.057***	-0.066***	-0.048*	-0.051*
	(0.018)	(0.020)	(0.019)	(0.024)	(0.025)	(0.027)
$\widehat{\theta}_\pi$	0.095	–	0.057	–	0.029	–
	(0.075)		(0.102)		(0.128)	
$\widehat{\theta}_p$	–	-0.008	–	-0.001	–	-0.008
		(0.042)		(0.052)		(0.057)
$N \times T$	1618	1641	1603	1626	1588	1611

Notes: The estimates are based on the following specification:

$$y_{it} = c_i + \boldsymbol{\theta}_i' \mathbf{x}_{it} + \sum_{\ell=0}^{p-1} \boldsymbol{\delta}_{i\ell}' \Delta \mathbf{x}_{i,t-\ell} + \omega_{iy} \bar{y}_t + \sum_{\ell=0}^{3} \boldsymbol{\omega}_{i,x\ell}' \bar{\mathbf{x}}_{t-\ell} + e_{it},$$

where in (i) y_{it} is the log of real GDP, $\mathbf{x}_{it} = (d_{it}, \pi_{it})'$, d_{it} is the log of the debt/GDP ratio, and π_{it} is the inflation rate and in (ii) y_{it} is the log of real GDP, $\mathbf{x}_{it} = (d_{it}, p_{it})'$, d_{it} is the log of the debt/GDP ratio, and p_{it} is the log of the CPI. Symbols ***, **, and * denote significance at 1%, 5%, and at 10% respectively.

Concluding Remarks

- To study the relationship between inflation-debt-growth, we adopted two econometric techniques: CS-ARDL and CS-DL approaches. They both consider different dynamics for each country and are consistent under both cross sectional dependence and cross country heterogeneity. The CS-DL approach is a new method for estimating the long-run effect *directly* in dynamic panel data models (not based on short-run estimates).
- We offered evidence on growth-reducing effects of persistently high inflation across countries and over time.
- We showed that the relationship between debt and growth is negative over a long time horizon.

Main Implications of Our Empirical Findings

- Our results suggest that if the debt level is increased and this increase is permanent, then it will have negative effects on economic growth in the long run.
- On the other hand, if the debt is increased (for instance to help smooth out business cycle fluctuations) and this increase is temporary, then there are no long-run negative effects on output growth.
- Our results also show that there is no simple common threshold for the *level* of government debt above which long-term growth is negatively affected. However, the threshold becomes important only at times of rising, rather falling debt. Provided that debt is on a downward path, a country with a high level of debt can grow just as fast as its peers.
- However, available data do not allow estimation of country-specific threshold levels of debt to GDP, and pooled estimates of a universal threshold could be misleading.