# Learning About Rare Disasters

## Implications for Consumption and Asset Prices

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# What We Do

- Minimal extension of the Mehra-Prescott-Rietz asset-pricing framework to an incomplete information setting
- Two key ingredients
  - variable growth persistence
    - Learning about growth persistence magnifies economic uncertainty
  - relaxed independence axiom
    - the recursive Epstein-Zin preferences configured so that early resolution of uncertainty is preferred.

# Endogenous Uncertainty Shocks

- countercyclical variation in the forecast-error variance

$$\text{Var}_t \{g_{t,T}\}$$

of the T-period consumption growth rate

$$g_{t,T} = \log\left(\frac{C_{t+T}}{C_t}\right)$$

**Fact**

*Our model endogenously generates*

$$Var_t \{g_{t,T}\} \quad Larger\ in\ Recessions\ compared\ to\ Expansions$$

# Endogenous Uncertainty Shocks (Cont'd)

- Matching monotonic patterns across the phases of the business cycle

**Fact**

*Our model endogenously generates*

$$Var_t\left\{g_{t,T}\right\} \quad \searrow \quad in\ Expansions$$

$$Var_t\left\{g_{t,T}\right\} \quad \nearrow \quad in\ Recessions$$

# Equity Prices

- Procyclical variation in
  - price-dividend ratios
- Countercyclical variation in
  - risk premiums
  - return volatility
  - Sharpe ratios
- These effects naturally induce
  - leverage effect
  - mean reversion of excess returns
  - predictability
    - excess return
    - consumption volatility

  from price-dividend ratio

# Equity Prices across Phases of Business Cycle

**Fact**

*Our model endogenously generates*

*Risk Premium*

*Return Volatility* $\nearrow$ *in Recessions*

*Sharpe Ratio*

# Real Bond Prices

- Matching the real yield curve
  - level
  - variability
  - persistence

  of yields

$$\text{Real(!) Yield Curve slopes} \searrow \text{in Expansions}$$

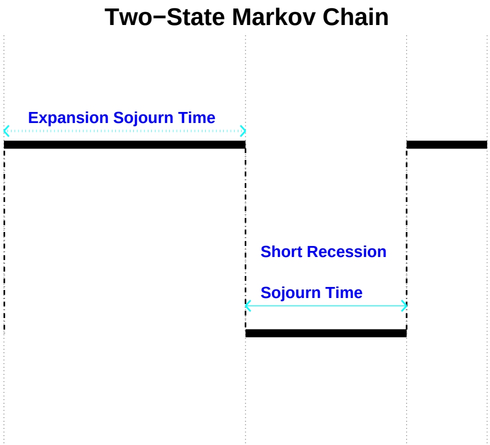
$$\text{Real(!) Yield Curve slopes} \nearrow \text{in Recessions}$$

- Matching bond risk premiums
  - level
  - variability
  - persistence

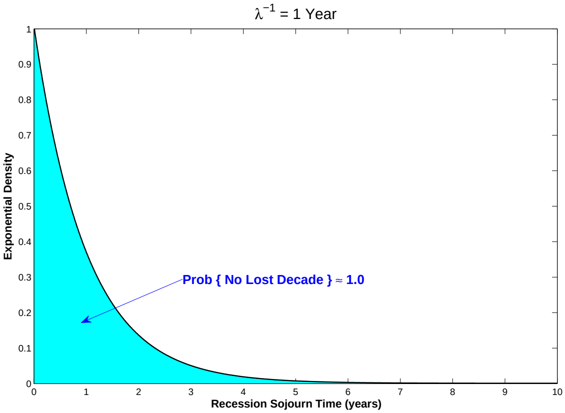
# Equity Option Prices

- Our preliminary results indicate
  - the implied volatility curves of S&P 500 index options in our model
    - mildly downward sloping
    - display negligible curvature

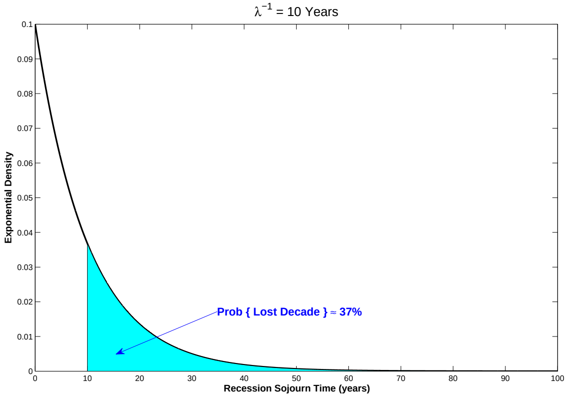
# Two-State Markov Chain



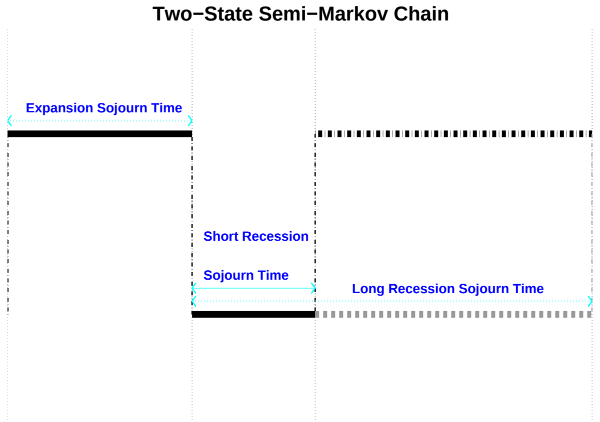
# Fast Tail Slimming



# Slow Tail Slimming



# Learning About Tail Slimming



# Standard Asset-Pricing Framework

- representative agent endowment economy á la Lucas (1978) and Mehra-Prescott (1985) with
  - recursive Epstein-Zin preferences

$$J_t = \left\{ e^{-\delta} C_t^{1-\frac{1}{\theta}} + \left(1 - e^{-\delta}\right) \left(E_t \left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\theta}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\theta}}}$$

where $\theta$ = elasticity of intertemporal substitution, $\gamma$ = relative risk aversion and $\delta$ = subjective discount rate

  - with hidden states of the macroeconomy $\Longrightarrow$ Bayesian learning

# Consumption Dynamics

- Subject the endowment growth rate

$$g_{t+1} = \log\left(\frac{C_{t+1}}{C_t}\right)$$

to hidden regime shifts

$$g_{t+1} = \underbrace{\mu(S_t)}_{\text{Predictable Component}} + \underbrace{\sigma^u \varepsilon_{t+1}}_{\text{Consumption Growth Surprise}}$$

- Consumption = Endowment in equilibrium

# Predictable Component $\mu(S_t)$ Specification

- Consider a three-state Markov chain

$$S_t \in \{0 = \text{Expansion}, 1 = \text{Recession}, 2 = \text{Lost Decade}\}$$

with the transition probability matrix

$$P = \begin{pmatrix} p_1 & q \times p_1 & (1-q) \times p_1 \\ 1 - p_2 & p_2 & 0 \\ 1 - p_3 & 0 & p_3 \end{pmatrix}$$

and consumption growth rates

$$\mu(1), \mu(2) \text{ and } \mu(3)$$

# Semi-Markov Property

## Consumption Growth Rates

**Fact**

*The growth rates in the recession and the lost decade are exactly equal,*

$$\mu(2) = \mu(3)$$

- Our maximum-likelihood estimates
  - $\mu(2) = \mu(3) \approx -0.79\%$ per year
- The model is best thought of as two-state semi-Markov model
  - the recession sojourn time is not exponentially distributed
  - the hazard rate of ending the recession is not constant

# Difference from Rietz (1988, JME)

**Fact**

*The growth rates in the recession and the lost decade are significantly different,*

$$\mu(2) \approx -0.79\% \textit{ per year}$$
$$\mu(3) \approx -79\% \textit{ per year}$$

- Moreover, recessions $S_t \in \{1, 2\}$ are not persistent

# Markov vs. Semi-Markov Chain

- Consider standard two-state Markov model
  - recessions last about one year, hence the hazard rate

$$\lambda = 1$$

  - probability of observing a lost decade is

$$P\left\{\text{Low-Growth Sojourn Time} > 10 \text{ years} \middle| S \neq \text{Expansion}\right\}$$
$$= \int_{10}^{\infty} \lambda e^{-\lambda\tau} \mathrm{d}\tau = \exp\left(-10 \times \lambda\right) = \exp\left(-10\right) = 0.00005$$

# Discriminating between Markov and Semi-Markov Specification

## Algorithm

*Bayesian Model Selection follows these steps:*

1. *Assume diffuse priors so the Prior Odds are*

$$\frac{\mathbb{P}\left\{Markov\ Model\ \right\}}{\mathbb{P}\left\{Semi\text{-}Markov\ Model\ \right\}} = 1$$

2. *Update using the Bayes Rule*
3. *Find the Posterior Odds*

$$\frac{\mathbb{P}\left\{\ Markov\ Model \mid Data\right\}}{\mathbb{P}\left\{\ Semi\text{-}Markov\ Model \mid Data\right\}}$$

# Consumption and Dividend Data Excluding Asset Prices

## Bayesian Model Selection

**Claim**

The posterior odds

$$\frac{\mathbb{P}\{\text{Markov Model} \mid \text{Data}\}}{\mathbb{P}\{\text{Semi-Markov Model} \mid \text{Data}\}} \approx 1$$

- The implications are the following

**Corollary**

*The time-series properties for consumption as well as dividends are similar.*

# Consumption and Dividend Data Including Asset Prices

## Bayesian Model Selection

### Claim

The posterior odds

$$\frac{\mathbb{P}\left\{\text{Markov Model} \mid \text{Data}\right\}}{\mathbb{P}\left\{\text{Semi-Markov Model} \mid \text{Data}\right\}} \ll 1$$

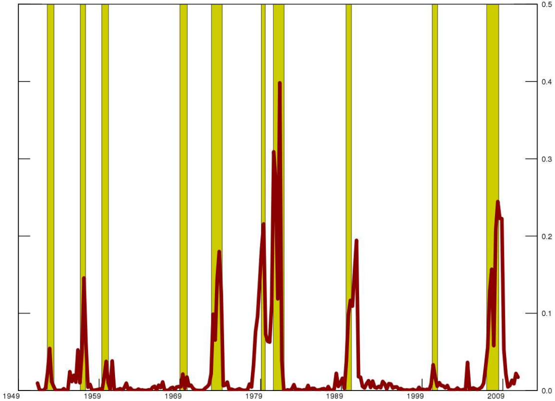
- The implications are the following

### Corollary

*The time-series properties for asset prices are very different.*

# Filtered Disaster Probability

$$\pi_3 = \mathbb{P}\{\text{Lost Decade} | \text{Historical Data}\}$$



# Cash-Flow Long-Run Averages

| | Annualized | | | Variance Ratios | | | |
|---|---|---|---|---|---|---|---|
| | Mean | S.D. | AC1 | 2 | 3 | 4 | 5 |
| **Data** | | | | Panel A: 1952:I–2011:IV | | | |
| Consumption | 1.89 | 1.26 | 0.01 | 1.42 | 1.68 | 1.06 | 1.64 |
| | (0.10) | (0.12) | (0.00) | (0.17) | (0.28) | (0.40) | (0.44) |
| Dividend | 2.06 | 10.38 | 0.01 | 1.33 | 0.94 | 1.10 | 1.25 |
| | (0.36) | (0.72) | (0.01) | (0.13) | (0.31) | (0.34) | (0.38) |
| **Model** | | | | Panel B: Monte Carlo Simulation | | | |
| Consumption | 1.87 | 1.29 | 0.00 | 1.40 | 1.72 | 1.98 | 2.21 |
| Dividend | 1.95 | 10.38 | 0.00 | 1.09 | 1.16 | 1.22 | 1.27 |

# Long-Run Averages for Equity and Bond Prices

Feeding Historical Beliefs from 1952:II to 2011:IV into our Model

| | Markov Model | | | Semi-Markov | | |
|---|---|---|---|---|---|---|
| | Mean | S.D. | AC1 | Mean | S.D. | AC1 |
| Levered Equity | | | | | | |
| Risk Premium | 0.90 | 0.44 | 0.26 | 5.58 | 3.52 | 0.23 |
| Volatility | 11.59 | 0.58 | 0.28 | 14.99 | 2.99 | 0.40 |
| Sharpe Ratio | 0.08 | 0.03 | 0.26 | 0.35 | 0.14 | 0.11 |
| Price-Dividend Ratio | 111.88 | 0.02 | 0.35 | 23.26 | 8.08 | 0.43 |
| Real Bond Prices | | | | | | |
| Short-Term Yield | 2.52 | 0.44 | 0.35 | 2.03 | 0.79 | 0.45 |
| Long-Term Yield | 2.39 | 0.02 | 0.35 | 0.83 | 0.08 | 0.45 |
| 30-Year Term | -0.13 | 0.72 | -0.03 | -1.30 | 2.46 | -0.09 |

# Averages Across the Expansions and Recessions

Consumption Growth Rate, Monte Carlo Results

| Conditional Moments | Annual Averages Across | |
|---|---|---|
| | Expansion | Recession |
| Conditional Mean | 2.08 | 1.04 |
| Conditional Volatility (Uncertainty Shocks) | 1.36 | 1.74 |

# Averages Across the Expansions and Recessions

Equity Prices, Monte Carlo Results

| Annual Moments | Expansion | Recession |
|---|---|---|
| Data (Lustig & Verdelhan, JME, 2013) | | |
| Mean Levered Return | 5.28 | 11.31 |
| | (1.87) | (2.20) |
| Levered Sharpe Ratio | 0.38 | 0.66 |
| | (0.14) | (0.14) |
| 2-State Semi-Markov Model | | |
| Mean Risk Premium | 5.52 | 8.88 |
| Conditional Volatility | 14.71 | 19.43 |
| Sharpe Ratio | 0.35 | 0.43 |

# Dynamics Across the Phases of the Expansions

Starting in $n$-th quarter after regime shift, Monte Carlo Results

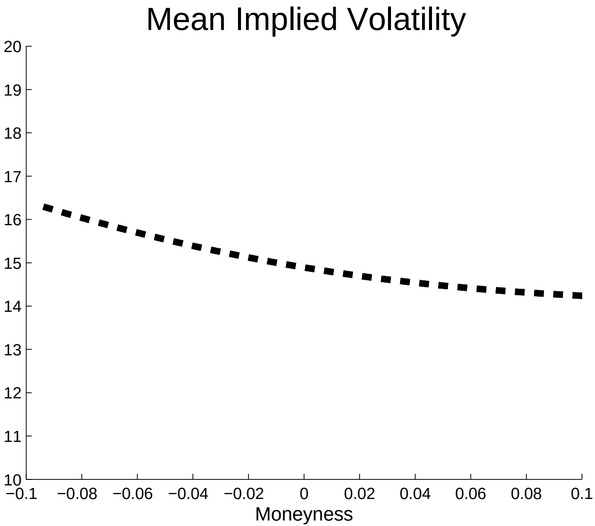
| Annual Moments | n=1 | n=2 | n=3 | n=4 | n=5 |
|---|---|---|---|---|---|
| Consumption Growth | | | | | |
| Mean | 1.34 | 1.84 | 2.01 | 2.06 | 2.07 |
| Volatility | 1.72 | 1.54 | 1.42 | 1.38 | 1.36 |
| Asset Data | | | | | |
| Mean Levered Return | 7.45 | 2.77 | 1.89 | 5.59 | 8.67 |
| Levered Sharpe Ratio | 0.51 | 0.19 | 0.14 | 0.42 | 0.67 |
| Model | | | | | |
| Mean Risk Premium | 10.96 | 10.55 | 8.98 | 7.53 | 6.50 |
| Conditional Volatility | 19.89 | 18.45 | 16.92 | 15.84 | 15.16 |
| Sharpe Ratio | 0.53 | 0.53 | 0.49 | 0.43 | 0.39 |

# Dynamics Across the Phases of the Recessions

Starting in $n$-th quarter after regime shift, Monte Carlo Results

| Annual Moments | n=1 | n=2 | n=3 | n=4 | n=5 |
|---|---|---|---|---|---|
| **Consumption Growth** | | | | | |
| Mean | 1.94 | 1.43 | 0.93 | 0.65 | 0.51 |
| Volatility | 1.47 | 1.72 | 1.78 | 1.79 | 1.79 |
| **Asset Data** | | | | | |
| Mean Levered Return | 7.53 | 14.13 | 11.45 | 12.96 | 10.49 |
| Levered Sharpe Ratio | 0.43 | 0.78 | 0.62 | 0.82 | 0.67 |
| **Model** | | | | | |
| Mean Risk Premium | 6.86 | 9.01 | 10.15 | 10.94 | 11.54 |
| Conditional Volatility | 15.87 | 18.11 | 19.54 | 20.52 | 21.24 |
| Sharpe Ratio | 0.41 | 0.49 | 0.51 | 0.53 | 0.54 |

# Fitting S&P 500 Option Prices



# Conclusion

- Our model is a minimal extension of the Mehra-Prescott-Rietz asset-pricing framework
- Our model introduces tail uncertainty about recession sojourn times
- Our model can explain the unconditional as well as conditional moments of
    - consumption
    - equity prices
    - real bond prices
    - equity option prices